11. We apply Eq. 4-10 and Eq. 4-16.

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\mathrm{i}} + 4t^2\,\hat{\mathrm{j}} + t\,\hat{\mathrm{k}}) = 8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}\,.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}\,(8t\,\hat{\mathrm{j}} + \hat{\mathrm{k}}) = 8\,\hat{\mathrm{j}}\,.$$

17. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is $v_y = v_{0y} + a_y t = -0.500t$ since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$\vec{r} = \left(3.00\hat{\mathrm{i}}\right)\left(3.00\right) + \frac{1}{2}\left(-1.00\,\hat{\mathrm{i}} - 0.50\,\hat{\mathrm{j}}\right)\left(3.00\right)^2 = (4.50\,\hat{\mathrm{i}} - 2.25\,\hat{\mathrm{j}}) \text{ m.}$$

29. The initial velocity has no vertical component — only an x component equal to +2.00 m/s. Also, $y_0 = +10.0$ m if the water surface is established as $y = 0$.

(a) $x - x_0 = v_x t$ readily yields $x - x_0 = 1.60$ m.

(b) Using $y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$, we obtain $y = 6.86$ m when $t = 0.800$ s and $v_{0y}=0$.

(c) Using the fact that y = 0 and y_0 = 10.0, the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ leads to $t = \sqrt{2(10.0\ \text{m})/9.80\ \text{m/s}^2} = 1.43\ \text{s}$. During this time, the x-displacement of the diver is $x - x_0$ = (2.00 m/s)(1.43 s) = 2.86 m.

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write θ_0 = –37.0° for the angle measured from +x, since the angle given in the problem is measured from the –y direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 4-22 to find v_0:

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \quad\Rightarrow\quad 0 - 730\ \text{m} = v_0 \sin(-37.0°)(5.00\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(5.00\ \text{s})^2$$

which yields v_0 = 202 m/s.

(b) The horizontal distance traveled is $x = v_0 t\cos\theta_0$ = (202 m/s)(5.00 s)cos(–37.0°) = 806 m.

(c) The x component of the velocity (just before impact) is

$$v_x = v_0\cos\theta_0 = (202\ \text{m/s})\cos(-37.0°) = 161\ \text{m/s}.$$

(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202\ \text{m/s}) \sin(-37.0°) - (9.80\ \text{m/s}^2)(5.00\ \text{s}) = -171\ \text{m/s}.$$

39. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 t\cos\theta_0$ and $0 = v_0 t\sin\theta_0 - \frac{1}{2}gt^2$. Eliminating t leads to $2v_0^2\sin\theta_0\cos\theta_0 - gd = 0$. Using $\sin\theta_0\cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \ \Rightarrow\ \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\ \text{m/s}^2)(45.7\ \text{m})}{(460\ \text{m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$ and consequently θ_0 = 0.0606°. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\ \text{m})\tan(0.0606°) = 0.0484\ \text{m} = 4.84\ \text{cm}.$$

47. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26.

(a) We want to know how high the ball is from the ground when it is at x = 97.5 m, which requires knowing the initial velocity. Using the range information and θ_0 = 45°, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(107 \text{ m})}{1}} = 32.4 \text{ m/s}.$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos \theta_0} = \frac{97.5 \text{ m}}{(32.4 \text{ m/s}) \cos 45°} = 4.26 \text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = 9.88 \text{ m}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At t = 4.26 s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

51. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s) θ_0 so that y = 3.44 m when x = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin \theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos \theta_0$, and when this is substituted into the second the result is

$$y = x \tan \theta_0 - \frac{gx^2}{2v_0^2 \cos^2 \theta_0}.$$

One may solve this by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity $1 / \cos^2 \theta_0 = 1 + \tan^2 \theta_0$, we obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for tan θ_0. To simplify writing the solution, we denote $c = \frac{1}{2}gx^2/v_0^2 = \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(50\text{ m}\right)^2/\left(25\text{ m/s}\right)^2 = 19.6\text{m}.$ Then the second-order equation becomes $c\tan^2\theta_0 - x\tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50\text{ m} \pm \sqrt{(50\text{ m})^2 - 4(3.44\text{ m} + 19.6\text{ m})(19.6\text{ m})}}{2(19.6\text{ m})}.$$

The two solutions are given by tan θ_0 = 1.95 and tan θ_0 = 0.605. The corresponding (first-quadrant) angles are θ_0 = 63° and θ_0 = 31°. Thus,

(a) The smallest elevation angle is θ_0 = 31°, and

(b) The greatest elevation angle is θ_0 = 63°.

If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

53. We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n - 1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction. The coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}}.$$

The x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52\text{ m/s})\sqrt{\frac{2n(0.203\text{ m})}{9.8\text{ m/s}^2}} = (0.309\text{ m})\sqrt{n}.$$

The method is to try values of n until we find one for which x/w is less than n but greater than $n - 1$. For $n = 1$, $x = 0.309$ m and $x/w = 1.52$, which is greater than n. For $n = 2$, $x = 0.437$ m and $x/w = 2.15$, which is also greater than n. For $n = 3$, $x = 0.535$ m and $x/w = 2.64$. Now, this is less than n and greater than $n - 1$, so the ball hits the third step.

67. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10 \text{ m})\sqrt{-\frac{9.8 \text{ m/s}^2}{2(-2.0 \text{ m})}} = 15.7 \text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7 \text{ m/s})^2}{1.5 \text{ m}} = 160 \text{ m/s}^2.$$

75. Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9 \text{ m/s}}{8.0 \text{ m/s}} = 1.74$$

which yields $\theta = 60°$.

77. Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop is $v_h = 30$ m/s, the same as the speed of the train. If v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then $\tan\theta = v_h/v_v$. Thus $v_v = v_h/\tan\theta = (30 \text{ m/s})/\tan 70° = 10.9$ m/s. The speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30 \text{ m/s})^2 + (10.9 \text{ m/s})^2} = 32 \text{ m/s}.$$

91. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the firing point, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

$$t=\sqrt{\frac{2(0.019\text{ m})}{9.8\text{ m/s}^2}}=6.2\times10^{-2}\text{ s}.$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since $x = 30$ m is the horizontal position of the target, we have $x = v_0t$. Thus,

$$v_0=\frac{x}{t}=\frac{30\text{ m}}{6.3\times10^{-2}\text{ s}}=4.8\times10^{2}\text{ m/s}.$$

107. (a) Eq. 2-15 can be applied to the vertical (y axis) motion related to reaching the maximum height (when $t = 3.0$ s and $v_y = 0$):

$$y_{\max}-y_0 = v_yt-\frac{1}{2}gt^2 .$$

With ground level chosen so $y_0 = 0$, this equation gives the result $y_{\max}=\frac{1}{2}g(3.0\text{ s})^2 = 44$ m.

(b) After the moment it reached maximum height, it is falling; at $t = 2.5$ s, it will have fallen an amount given by Eq. 2-18

$$y_{\text{fence}}-y_{\max} = (0)(2.5\text{ s})-\frac{1}{2}g(2.5\text{ s})^2$$

which leads to $y_{\text{fence}} = 13$ m.

(c) Either the *range* formula, Eq. 4-26, can be used or one can note that after passing the fence, it will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find the original x-component from 97.5 m = v_{0x}(5.5 s) and then apply it to that final 0.5 s. Thus, we find $v_{0x} = 17.7$ m/s and that after the fence

$$\Delta x = (17.7\text{ m/s})(0.5\text{ s}) = 8.9\text{ m}.$$

111. Since the x and y components of the acceleration are constants, we can use Table 2-1 for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) Since $\vec{r}_0 = 0$, the position vector of the particle is (adapting Eq. 2-15)

$$\vec{r}=\vec{v}_0t+\frac{1}{2}\vec{a}t^2=\left(8.0\,\hat{\text{j}}\right)t+\frac{1}{2}\left(4.0\,\hat{\text{i}}+2.0\,\hat{\text{j}}\right)t^2=\left(2.0t^2\right)\hat{\text{i}}+\left(8.0t+1.0t^2\right)\hat{\text{j}}.$$

Therefore, we find when x = 29 m, by solving $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is y = (8.0 m/s)(3.8 s) + (1.0 m/s^2)(3.8 s)^2 = 45 m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t.$$

Thus, at t = 3.8 s, the velocity is

$$\vec{v} = (8.0\text{ m/s})\,\hat{\text{j}} + \left((4.0\text{ m/s}^2)\,\hat{\text{i}} + (2.0\text{ m/s}^2)\,\hat{\text{j}}\right)\left(3.8\text{ s}\right) = (15.2\text{ m/s})\,\hat{\text{i}} + (15.6\text{ m/s})\,\hat{\text{j}}$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2\text{ m/s})^2 + (15.6\text{ m/s})^2} = 22\text{ m/s}.$$

121. On the one hand, we could perform the vector addition of the displacements with a vector-capable calculator in polar mode $((75 \angle 37°) + (65 \angle -90°) = (63 \angle -18°))$, but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with $+x$ East and $+y$ North. Lengths are in kilometers and times are in hours.

(a) We perform the vector addition of individual displacements to find the net displacement of the camel.

$$\Delta\vec{r}_1 = (75\text{ km})\cos(37°)\,\hat{\text{i}} + (75\text{ km})\sin(37°)\,\hat{\text{j}}$$
$$\Delta\vec{r}_2 = (-65\text{ km})\,\hat{\text{j}}$$
$$\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 = (60\text{ km})\,\hat{\text{i}} - (20\text{ km})\,\hat{\text{j}}\ .$$

If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length $|\Delta\vec{r}| = \sqrt{(60\text{ km})^2 + (-20\text{ km})^2} = 63\text{ km}$.

(b) The direction of $\Delta\vec{r}$ is $\theta = \tan^{-1}[(-20\text{ km})/(60\text{ km})] = -18°$, or $18°$ south of east.

(c) We use the result from part (a) in Eq. 4-8 along with the fact that Δt = 90 h. In unit vector notation, we obtain

$$\vec{v}_{\text{avg}} = \frac{(60\,\hat{\text{i}} - 20\,\hat{\text{j}})\text{ km}}{90\text{ h}} = (0.67\,\hat{\text{i}} - 0.22\,\hat{\text{j}})\text{ km/h}.$$

This leads to $|\vec{v}_{\text{avg}}| = 0.70\text{ km/h}$.

(d) The direction of $\vec{v}_{avg}$ is $\theta = \tan^{-1}[(-0.22 \text{ km/h})/(0.67 \text{ km/h})] = -18°$, or $18°$ south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain (140 km)/(90 h) = 1.56 km/h ≈ 1.6 km/h .

(f) The net displacement is required to be the 90 km East from *A* to *B*. The displacement from the resting place to *B* is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90 \text{ km})\,\hat{\text{i}}$$

which produces $\Delta\vec{r}_3 = (30 \text{ km})\hat{\text{i}} + (20 \text{ km})\hat{\text{j}}$ in unit-vector notation, or $(36 \angle 33°)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{\text{avg}}| = \frac{36\,\text{km}}{(120 - 90)\text{ h}} = 1.2\,\text{km/h}.$$

(g) The direction of $\vec{v}_{avg}$ is the same as $\vec{r}_3$ (that is, 33° north of east).