CHAPTER 5

Stresses in Beams (Basic Topics)

Beams are essential load carrying components in a wide variety of modern structures.

Chapter Objectives

- Develop a relationship between moment and curvature for a beam loaded by transverse applied loads and bending moments.
- Define the flexure formula, which shows that normal stresses vary linearly over the depth of a beam and are proportional to the bending moment and inversely proportional to the moment of inertia of the cross section.
- Define the section modulus of a beam and use it to design beams made of steel, wood, or other materials based upon an allowable stress for the material.
- Investigate shear stresses in beams of different shapes and study the variation of shear stress over the depth of a beam using a shear formula.
- Design the glued or nailed connections between the parts of built-up beams to ensure that the connections are strong enough to transmit the horizontal shear forces acting between the parts of the beam.
- Superpose bending and axial stresses for structural members subjected to simultaneous action of transverse and axial loads.
- Evaluate normal stresses in beams at locations of holes or other abrupt changes in cross section where stress concentrations occur.

Chapter Outline

5.1 Introduction 446
5.2 Pure Bending and Nonuniform Bending 446
5.3 Curvature of a Beam 447
5.4 Longitudinal Strains in Beams 449
5.5 Normal Stress in Beams (Linearly Elastic Materials) 453
5.6 Design of Beams for Bending Stresses 466
5.7 Nonprismatic Beams 476
5.8 Shear Stresses in Beams of Rectangular Cross Section 480
5.9 Shear Stresses in Beams of Circular Cross Section 488
5.10 Shear Stresses in the Webs of Beams with Flanges 491
***5.11** Built-Up Beams and Shear Flow 498
***5.12** Beams with Axial Loads 502
***5.13** Stress Concentrations in Bending 509
Chapter Summary & Review 514
Problems 518

***Specialized and/or advanced topics**

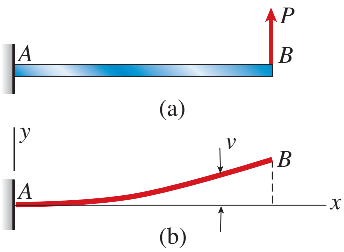
5.1 Introduction

In the preceding chapter, you saw how the loads acting on a beam create internal actions (or *stress resultants*) in the form of shear forces and bending moments. This chapter goes one step further and investigates the *stresses* and *strains* associated with those shear forces and bending moments. Knowing the stresses and strains, you will analyze and design beams subjected to a variety of loading conditions.

The loads acting on a beam cause the beam to bend (or *flex*), thereby deforming its axis into a curve. As an example, consider a cantilever beam AB subjected to a load P at the free end (Fig. 5-1a). The initially straight axis is bent into a curve (Fig. 5-1b), called the **deflection curve** of the beam.

FIGURE 5-1

Bending of a cantilever beam: (a) beam with load and (b) deflection curve



For reference purposes, a system of **coordinate axes** (Fig. 5-1b) is constructed with the origin located at a suitable point on the longitudinal axis of the beam. In this illustration, the origin is placed at the fixed support. The positive x axis is directed to the right, and the positive y axis is directed upward. The z axis, not shown in the figure, is directed outward (that is, toward the viewer), so that the three axes form a right-handed coordinate system.

The beams considered in this chapter are assumed to be symmetric about the x-y plane, which means that the y axis is an axis of symmetry of the cross section. In addition, all loads must act in the x-y plane. As a consequence, the bending deflections occur in this same plane, known as the **plane of bending**. Thus, the deflection curve shown in Fig. 5-1b is a plane curve lying in the plane of bending.

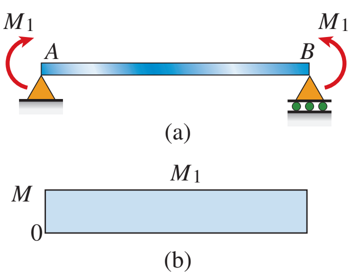
The **deflection** of the beam at any point along its axis is the *displacement* of that point from its original position measured in the y direction. Denote the deflection by the letter v to distinguish it from the coordinate y itself (see Fig. 5-1b).[1]

5.2 Pure Bending and Nonuniform Bending

When analyzing beams, it is often necessary to distinguish between pure bending and nonuniform bending. **Pure bending** refers to the flexure of a beam under a constant bending moment. Therefore, pure bending occurs only in regions of a beam where the shear force is zero (because $V = dM/dx$). In contrast, **nonuniform bending** refers to flexure in the presence of shear forces, which means that the bending moment changes as you move along the axis of the beam.

FIGURE 5-2

Simple beam in pure bending ($M = M_1$)



As an example of pure bending, consider a simple beam AB loaded by two couples M_1 having the same magnitude but acting in opposite directions (Fig. 5-2a). These loads produce a constant bending moment $M = M_1$ throughout the length of the beam, as shown by the bending moment diagram in (Fig. 5-2b). Note that the shear force V is zero at all cross sections of the beam.

Figure 5-3a shows pure bending, where the cantilever beam AB is subjected to a clockwise couple M_2 at the free end. There are no shear forces in this beam, and the bending moment M is constant throughout its length. The bending moment is negative ($M = -M_2$), as shown by the bending moment diagram in Fig. 5-3b.

The symmetrically loaded simple beam of Fig. 5-4a is an example of a beam that is partly in pure bending and partly in nonuniform bending, as seen from the shear-force and bending-moment diagrams (Figs. 5-4b and c).

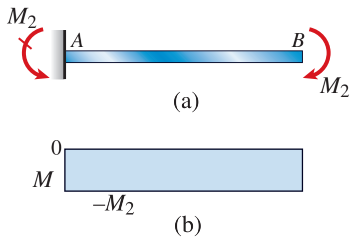
[1]In applied mechanics, the traditional symbols for displacements in the x, y, and z directions are u, v, and w, respectively.

The central region of the beam is in pure bending because the shear force is zero and the bending moment is constant. The parts of the beam near the ends are in nonuniform bending because shear forces are present and the bending moments vary.

In the following two sections, the strains and stresses in beams subjected only to pure bending are investigated. Fortunately, the results obtained for pure bending can be used even when shear forces are present, as explained in Section 5.8.

FIGURE 5-3

Cantilever beam in pure bending ($M = -M_2$)



5.3 Curvature of a Beam

When loads are applied to a beam, its longitudinal axis is deformed into a curve, as illustrated previously in Fig. 5-1. The resulting strains and stresses in the beam are directly related to the **curvature** of the deflection curve.

To illustrate the concept of curvature, consider again a cantilever beam subjected to a load P acting at the free end (see Fig. 5-5a on the next page). The deflection curve of this beam is shown in Fig. 5-5b. For purposes of analysis, identify two points m_1 and m_2 on the deflection curve. Point m_1 is selected at an arbitrary distance x from the y axis, and point m_2 is located a small distance ds further along the curve. At each of these points, draw a line normal to the *tangent* to the deflection curve, that is, normal to the curve itself. These normals intersect at point O', which is the **center of curvature** of the deflection curve. Because most beams have very small deflections and nearly flat deflection curves, point O' is usually located much farther from the beam than is indicated in the figure.

The distance m_1O' from the curve to the center of curvature is called the **radius of curvature** ρ (rho), and the **curvature** κ (kappa) is defined as the reciprocal of the radius of curvature. Thus,

$$\kappa = \frac{1}{\rho} \tag{5-1}$$

Curvature is a measure of how sharply a beam is bent. If the load on a beam is small, the beam will be nearly straight, the radius of curvature will be very large, and the curvature will be very small. If the load is increased, the amount of bending will increase—the radius of curvature will become smaller, and the curvature will become larger.

The geometry of triangle $O'm_1m_2$ (Fig. 5-5b) leads to

$$\rho d\theta = ds \tag{5-2}$$

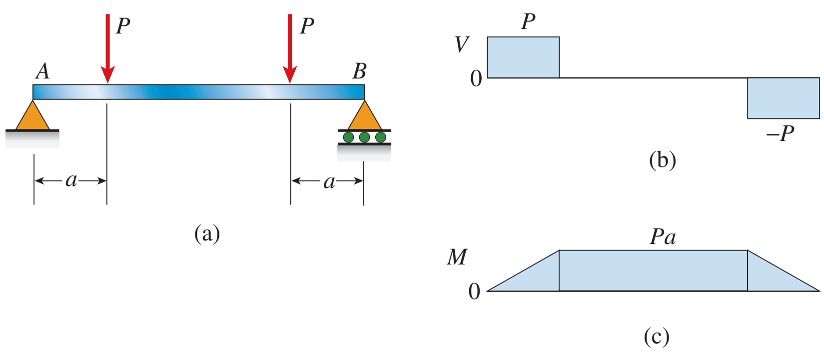
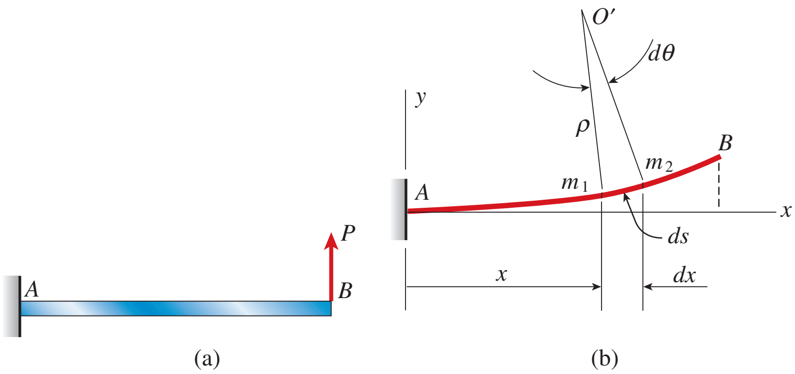


FIGURE 5-4

Simple beam with central region in pure bending and end regions in nonuniform bending

FIGURE 5-5

Curvature of a bent beam: (a) beam with load and (b) deflection curve



in which $d\theta$ (measured in radians) is the infinitesimal angle between the normals and ds is the infinitesimal distance along the curve between points m_1 and m_2. Combine Eq. (5-2) with Eq. (5-1) to get

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \tag{5-3}$$

This equation for **curvature** is derived in textbooks on calculus and holds for any curve, regardless of the amount of curvature. If the curvature is *constant* throughout the length of a curve, the radius of curvature also will be constant, and the curve will be an arc of a circle.

The deflections of a beam are usually very small compared to its length (consider, for instance, the deflections of the structural frame of an automobile or a beam in a building). Small deflections mean that the deflection curve is nearly flat. Consequently, the distance ds along the curve may be set equal to its horizontal projection dx (see Fig. 5-5b). Under these special conditions of **small deflections**, the equation for the curvature becomes

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{dx} \tag{5-4}$$

Both the curvature and the radius of curvature are functions of the distance x measured along the x axis. It follows that the position O' of the center of curvature also depends upon the distance x.

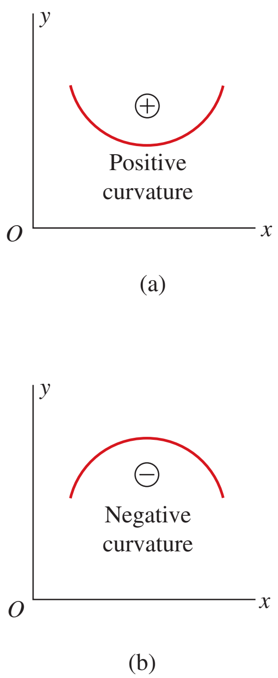
The curvature at a particular point on the axis of a beam depends upon the bending moment at that point and upon the properties of the beam itself (shape of cross section and type of material). Therefore, if the beam is prismatic and the material is homogeneous, the curvature varies only with the bending moment (see Section 5.5). Consequently, a beam in *pure bending* has constant curvature, and a beam in *nonuniform bending* has varying curvature.

The **sign convention for curvature** depends upon the orientation of the coordinate axes. If the x axis is positive to the right and the y axis is positive upward, as shown in Fig. 5-6, then the curvature is positive when the beam is bent concave upward and the center of curvature is above the beam. Conversely, the curvature is negative when the beam is bent concave downward, and the center of curvature is below the beam.

The next section shows how the longitudinal strains in a bent beam are determined from its curvature, and Chapter 9 covers how curvature is related to the deflections of beams.

FIGURE 5-6

Sign convention for curvature



5.4 Longitudinal Strains in Beams

The longitudinal strains in a beam can be found by analyzing the curvature of the beam and the associated deformations. For this purpose, consider a portion *AB* of a beam in pure bending subjected to positive bending moments *M* (Fig. 5-7a). Assume that the beam initially has a straight longitudinal axis (the *x* axis in the figure) and that its cross section is symmetric about the *y* axis, as shown in Fig. 5-7b.

Under the action of the bending moments, the beam deflects in the *x-y* plane (the plane of bending) and its longitudinal axis is bent into a circular curve (curve *s–s* in Fig. 5-7c). The beam is bent concave upward, which is positive curvature (Fig. 5-6a).

Cross sections of the beam, such as sections *mn* and *pq* in Fig. 5-7a, remain plane and normal to the longitudinal axis (Fig. 5-7c). The fact that cross sections of a beam in pure bending remain plane is so fundamental to beam theory that it is often called an assumption. However, it also could be called a theorem because it can be proved rigorously using only rational arguments based upon symmetry (Ref. 5-1). The basic point is that the symmetry of the beam and its loading (Figs. 5-7a and b) means that all elements of the beam (such as element *mpqn*) must deform in an identical manner, which is possible only if cross sections remain plane during bending (Fig. 5-7c). This conclusion is valid for beams of any material, whether the material is elastic or inelastic, linear or nonlinear. Of course, the material properties, like the dimensions, must be symmetric about the plane of bending. *Note:* Even though a plane cross section in pure bending remains plane, there still may be deformations in the plane

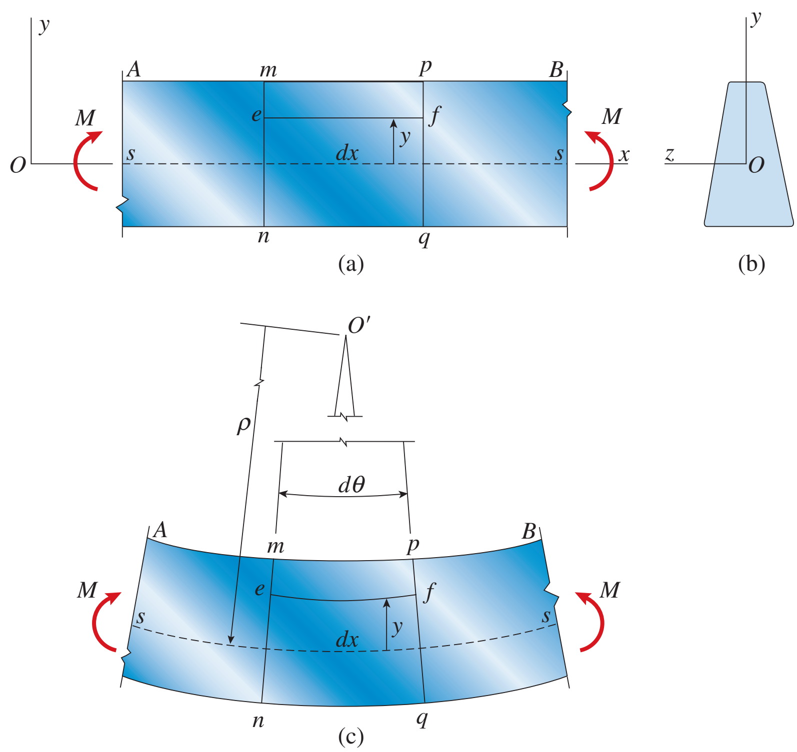


FIGURE 5-7

Deformations of a beam in pure bending: (a) side view of beam, (b) cross section of beam, and (c) deformed beam itself. Such deformations are due to the effects of Poisson's ratio, as explained at the end of this discussion.

Because of the bending deformations shown in Fig. 5-7c, cross sections *mn* and *pq* rotate with respect to each other about axes perpendicular to the *x*-*y* plane. Longitudinal lines on the lower part of the beam are elongated, whereas those on the upper part are shortened. Thus, the lower part of the beam is in tension and the upper part is in compression. Somewhere between the top and bottom of the beam is a surface in which longitudinal lines do not change in length. This surface, indicated by the dashed line *s–s* in Figs. 5-7a and c, is called the **neutral surface** of the beam. Its intersection with any cross-sectional plane is called the **neutral axis** of the cross section; for instance, the *z* axis is the neutral axis for the cross section of Fig. 5-7b.

The planes containing cross sections *mn* and *pq* in the deformed beam (Fig. 5-7c) intersect in a line through the center of curvature O'. The angle between these planes is denoted $d\theta$, and the distance from O' to the neutral surface *s–s* is the radius of curvature ρ. The initial distance dx between the two planes (Fig. 5-7a) is unchanged at the neutral surface (Fig. 5-7c), hence $\rho d\theta = dx$. However, all other longitudinal lines between the two planes either lengthen or shorten, thereby creating **normal strains** ε_x.

To evaluate these normal strains, consider a typical longitudinal line *ef* located within the beam between planes *mn* and *pq* (Fig. 5-7a). Identify line *ef* by its distance *y* from the neutral surface in the initially straight beam. Now assume that the *x* axis lies along the neutral surface of the *undeformed* beam. Of course, when the beam deflects, the neutral surface moves with the beam, but the *x* axis remains fixed in position. Nevertheless, the longitudinal line *ef* in the deflected beam (Fig. 5-7c) is still located at the same distance *y* from the neutral surface. Thus, the length L_1 of line *ef* after bending takes place is

$$L_1 = (\rho - y)d\theta = dx - \frac{y}{\rho}dx$$

after substitution of $d\theta = dx/\rho$.

Since the original length of line *ef* is dx, it follows that its elongation is $L_1 - dx$, or $-ydx/\rho$. The corresponding *longitudinal strain* is equal to the elongation divided by the initial length dx; therefore, the **strain-curvature relation** is

$$\varepsilon_x = -\frac{y}{\rho} = -\kappa y \qquad \textbf{(5-5)}$$

where κ is the curvature [see Eq. (5-1)].

The preceding equation shows that the longitudinal strains in the beam are proportional to the curvature and vary linearly with the distance *y* from the neutral surface. When the point under consideration is above the neutral surface, the distance *y* is positive. If the curvature is also positive (as in Fig. 5-7c), then ε_x will be a negative strain, representing a shortening. By contrast, if the point under consideration is below the neutral surface, the distance *y* will be negative and, if the curvature is positive, the strain ε_x also will be positive, representing an elongation. Note that the **sign convention** for ε_x is the same as that used for normal strains in earlier chapters, namely, elongation is positive and shortening is negative.

Equation (5-5) for the normal strains in a beam was derived solely from the geometry of the deformed beam—the properties of the material did not enter into the discussion. Therefore, *the strains in a beam in pure bending vary linearly with distance from the neutral surface regardless of the shape of the stress-strain curve of the material.*

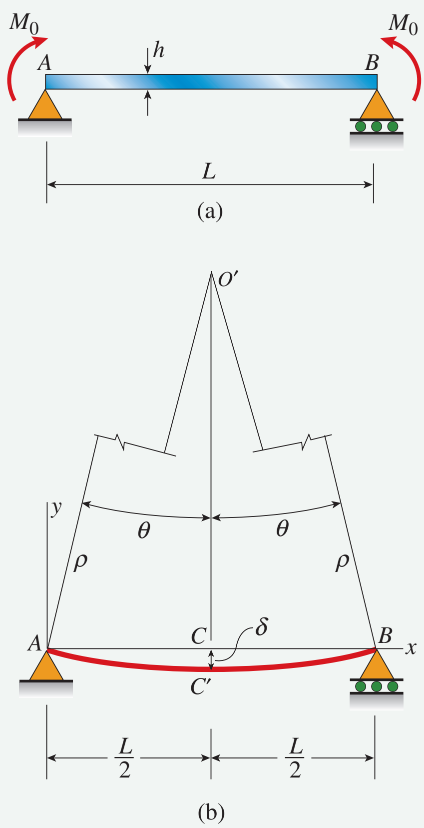
The next step in the analysis, namely, finding the stresses from the strains, requires the use of the *stress-strain curve*. This step is described in the next section for linearly elastic materials and in Section 6.10 for elastoplastic materials.

The longitudinal strains in a beam are accompanied by *transverse strains* (that is, normal strains in the y and z directions) because of the effects of Poisson's ratio. However, there are no accompanying transverse stresses because beams are free to deform laterally. This stress condition is analogous to that of a prismatic bar in tension or compression, and therefore, *longitudinal elements in a beam in pure bending are in a state of uniaxial stress.*

Example 5-1

FIGURE 5-8

Example 5-1: Beam in pure bending: (a) beam with loads and (b) deflection curve



A simply supported steel beam AB (Fig. 5-8a) of a length $L = 8.0$ ft and height $h = 6.0$ in. is bent by couples M_0 into a circular arc with a downward deflection δ at the midpoint (Fig. 5-8b). The longitudinal normal strain (elongation) on the bottom surface of the beam is 0.00125, and the distance from the neutral surface to the bottom surface of the beam is 3.0 in.

Determine the radius of curvature ρ, the curvature κ, and the deflection δ of the beam.

Note: This beam has a relatively large deflection because its length is large compared to its height ($L/h = 16$), and the strain of 0.00125 is also large. (This is about the same as the yield strain for ordinary structural steel.)

Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

Part (a): Curvature.

1, 2. ***Conceptualize*** [*hypothesize, sketch*], ***Categorize*** [*simplify, classify*]: Since the longitudinal strain at the bottom surface of the beam ($\varepsilon_x = 0.00125$) and the distance from the neutral surface to the bottom surface ($y = -3.0$ in.) are known, use Eq. (5-5) to calculate both the radius of curvature and the curvature.

3. ***Analyze*** [*evaluate; select relevant equations, carry out mathematical solution*]: Rearrange Eq. (5-5) and substitute numerical values to get

$$\rho = -\frac{y}{\varepsilon_x} = \frac{-3.0 \text{ in.}}{0.00125} = 2400 \text{ in.} = 200 \text{ ft} \quad \kappa = \frac{1}{\rho} = 0.0050 \text{ ft}^{-1}$$

4. ***Finalize*** [*conclude; examine answer—Does it make sense? Are units correct? How does it compare to similar problem solutions?*]: These results show that the radius of curvature is extremely large compared to the length of the beam even when the strain in the material is large. If, as usual, the strain is less, the radius of curvature is even larger.

Part (b): Deflection.

1, 2. ***Conceptualize, Categorize***: As pointed out in Section 5.3, a constant bending moment (pure bending) produces constant curvature throughout the length of a beam. Therefore, the deflection curve is a circular arc. From Fig. 5-8b, the distance from the center of curvature O' to the midpoint C' of the deflected beam is the radius of curvature ρ, and the distance from O' to point C on the x axis is $\rho \cos \theta$, where θ is angle $BO'C$. This leads to the expression for the deflection at the midpoint of the beam:

$$\delta = \rho(1 - \cos \theta) \tag{5-6}$$

For a nearly flat curve, assume that the distance between supports is the same as the length of the beam itself. Therefore, from triangle $BO'C$,

$$\sin \theta = \frac{L/2}{\rho} \tag{5-7}$$

3. ***Analyze***: Substitute numerical values to obtain

$$\sin \theta = \frac{(8.0 \text{ ft})(12 \text{ in./ft})}{2(2400 \text{ in.})} = 0.0200$$

and

$$\theta = 0.0200 \text{ rad} = 1.146°$$

For practical purposes, consider $\sin \theta$ and θ (radians) to be equal numerically because θ is a very small angle.

Now substitute into Eq. (5-6) for the deflection and obtain

$$\delta = \rho(1 - \cos \theta) = (2400 \text{ in.})(1 - 0.999800) = 0.480 \text{ in.}$$

4. ***Finalize***: This deflection is very small compared to the length of the beam, as shown by the ratio of the span length to the deflection:

$$\frac{L}{\delta} = \frac{(8.0 \text{ ft})(12 \text{ in./ft})}{0.480 \text{ in.}} = 200$$

This confirms that the deflection curve is nearly flat in spite of the large strains. Of course, in Fig. 5-8b, the deflection of the beam is highly exaggerated for clarity.

Note: The purpose of this example is to show the relative magnitudes of the radius of curvature, length of the beam, and deflection of the beam. However, the method used for finding the deflection has little practical value because it is limited to pure bending, which produces a circular deflected shape. More useful methods for finding beam deflections are presented in Chapter 9.

5.5 Normal Stresses in Beams (Linearly Elastic Materials)

Longitudinal strains ε_x in a beam in pure bending were investigated in the preceding section [see Eq. (5-5) and Fig. 5-7]. Since longitudinal elements of a beam are subjected only to tension or compression, now use the **stress-strain curve** for the material to determine the stresses from the strains. The stresses act over the entire cross section of the beam and vary in intensity, depending upon the shape of the stress-strain diagram and the dimensions of the cross section. Since the x direction is longitudinal (Fig. 5-7a), use the symbol σ_x to denote these stresses.

The most common stress-strain relationship encountered in engineering is the equation for a **linearly elastic material**. For such materials, substitute Hooke's law for uniaxial stress ($\sigma = E\varepsilon$) into Eq. (5-5) and obtain

$$\sigma_x = E\varepsilon_x = -\frac{Ey}{\rho} = -E\kappa y \qquad \textbf{(5-8)}$$

This equation shows that the normal stresses acting on the cross section vary linearly with the distance y from the neutral surface. This stress distribution is pictured in Fig. 5-9a for the case in which the bending moment M is positive and the beam bends with positive curvature.

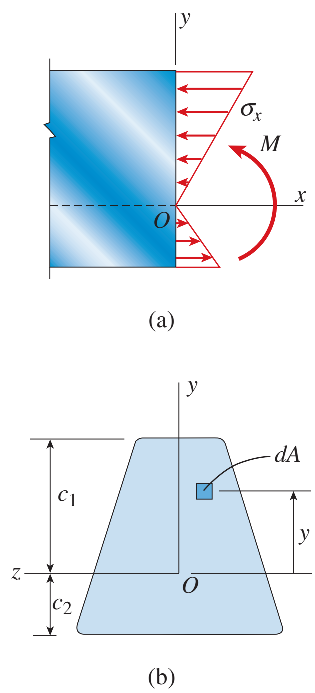
When the curvature is positive, the stresses σ_x are negative (compression) above the neutral surface and positive (tension) below it. In the figure, compressive stresses are indicated by arrows pointing *toward* the cross section and tensile stresses are indicated by arrows pointing *away* from the cross section.

In order for Eq. (5-8) to be of practical value, locate the origin of the coordinates so that you can determine the distance y. In other words, locate the neutral axis of the cross section. You also need to obtain a relationship between the curvature and the bending moment—so that you can substitute into Eq. (5-8) and obtain an equation relating the stresses to the bending moment. These two objectives can be accomplished by determining the resultant of the stresses σ_x acting on the cross section.

In general, the **resultant of the normal stresses** consists of two stress resultants: (1) a force acting in the x direction and (2) a bending couple acting about the z axis. However, the axial force is zero when a beam is in pure bending. Therefore, write the following equations of statics: (1) The resultant force in the x direction is equal to zero, and (2) the resultant moment is equal to the bending moment M. The first equation gives the location of the neutral axis, and the second gives the moment-curvature relationship.

FIGURE 5-9

Normal stresses in a beam of linearly elastic material: (a) side view of beam showing distribution of normal stresses and (b) cross section of beam showing the z axis as the neutral axis of the cross section



Location of Neutral Axis

To obtain the first equation of statics, consider an element of area dA in the cross section (Fig. 5-9b). The element is located at a distance y from the neutral axis; therefore, the stress σ_x acting on the element is given by Eq. (5-8). The *force* acting on the element is equal to $\sigma_x dA$ and is compressive when y is positive. Because there is no resultant force acting on the cross section, the integral of $\sigma_x dA$ over the area A of the entire cross section must vanish; thus, the *first equation of statics* is

$$\int_A \sigma_x dA = -\int_A E\kappa y\, dA = 0 \tag{5-9a}$$

Because the curvature κ and modulus of elasticity E are nonzero constants at any given cross section of a bent beam, they are not involved in the integration over the cross-sectional area. Therefore, drop them from the equation and obtain

$$\int_A y\, dA = 0 \tag{5-9b}$$

This equation states that the first moment of the area of the cross section, when evaluated with respect to the z axis, is zero. In other words, the z axis must pass through the centroid of the cross section.[2]

The z axis is also the neutral axis, so

The neutral axis passes through the centroid of the cross-sectional area when the material follows Hooke's law and there is no axial force acting on the cross section.

This observation makes it relatively simple to determine the position of the neutral axis.

As explained in Section 5.1, this discussion is limited to beams for which the y axis is an axis of symmetry. Consequently, the y axis also passes through the centroid. Therefore,

The origin O of coordinates (Fig. 5-9b) is located at the centroid of the cross-sectional area.

Because the y axis is an axis of symmetry of the cross section, the y axis is a *principal axis* (see Appendix D, Section D.8, for a discussion of principal axes). Since the z axis is perpendicular to the y axis, it too is a principal axis. Thus, when a beam of linearly elastic material is subjected to pure bending, *the y and z axes are principal centroidal axes.*

Moment-Curvature Relationship

The *second equation of statics* expresses the fact that the moment resultant of the normal stresses σ_x acting over the cross section is equal to the bending moment M (Fig. 5-9a). The element of force $\sigma_x dA$ acting on the element of area dA (Fig. 5-9b) is in the positive direction of the x axis when σ_x is positive and in the negative direction when σ_x is negative. Since the element dA is located above the neutral axis, a positive stress σ_x acting on that element produces an element of moment equal to $\sigma_x y dA$. This element of moment acts opposite in direction to the positive bending moment M shown in Fig. 5-9a. Therefore, the elemental moment is

$$dM = -\sigma_x y dA$$

[2]Centroids and first moments of areas are discussed in Appendix D, Sections D.1 and D.2.

The integral of all such elemental moments over the entire cross-sectional area A must equal the bending moment:

$$M = -\int_A \sigma_x y\,dA \tag{5-10a}$$

or, upon substituting for σ_x from Eq. (5-9),

$$M = \int_A \kappa E y^2 dA = \kappa E \int_A y^2 dA \tag{5-10b}$$

This equation relates the curvature of the beam to the bending moment M.

Since the integral in the preceding equation is a property of the cross-sectional area, it is convenient to rewrite the equation as

$$M = \kappa EI \tag{5-11}$$

in which

$$I = \int_A y^2 dA \tag{5-12}$$

This integral is the **moment of inertia** of the cross-sectional area with respect to the z axis (that is, with respect to the neutral axis). Moments of inertia are always positive and have dimensions of length to the fourth power; for instance, typical USCS units are in^4 and typical SI units are mm^4 when performing beam calculations.[3]

Equation (5-11) now can be rearranged to express the *curvature* in terms of the bending moment in the beam:

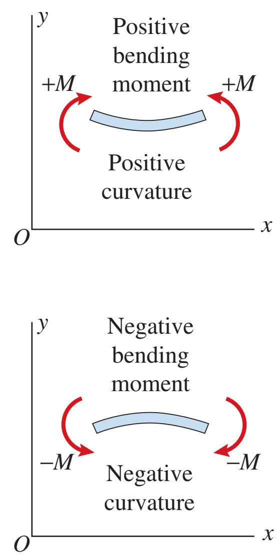
$$\kappa = \frac{1}{\rho} = \frac{M}{EI} \tag{5-13}$$

Known as the **moment-curvature equation**, Eq. (5-13) shows that the curvature is directly proportional to the bending moment M and inversely proportional to the quantity EI, which is called the **flexural rigidity** of the beam. Flexural rigidity is a measure of the resistance of a beam to bending, that is, the larger the flexural rigidity, the smaller the curvature for a given bending moment.

Comparing the **sign convention** for bending moments (Fig. 4-19) with that for curvature (Fig. 5-6), note that *a positive bending moment produces positive curvature and a negative bending moment produces negative curvature* (see Fig. 5-10).

FIGURE 5-10

Relationships between signs of bending moments and signs of curvatures



Flexure Formula

Now that the neutral axis has been located and the moment-curvature relationship has been derived, determine the stresses in terms of the bending moment. Substitute the expression for curvature [Eq. (5-13)] into the expression for the stress σ_x [Eq. (5-8)] to get

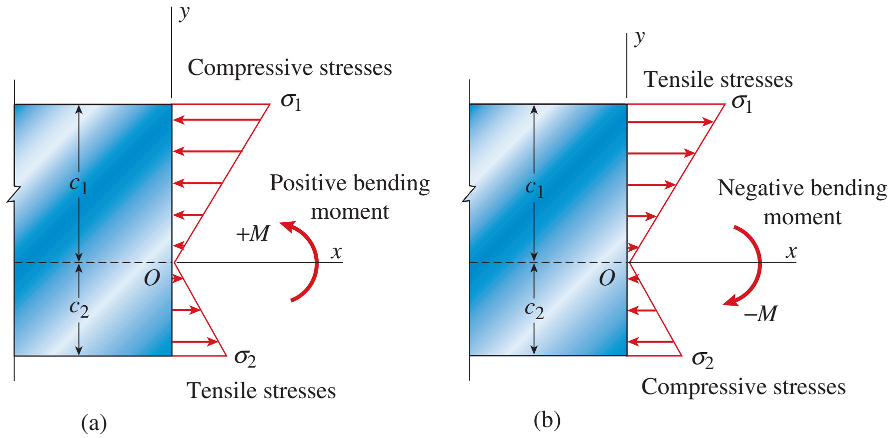
$$\sigma_x = -\frac{My}{I} \tag{5-14}$$

This equation, called the **flexure formula**, shows that the stresses are directly proportional to the bending moment M and inversely proportional to the moment of inertia I of the cross section. Also, the stresses vary linearly with the distance y from the neutral axis, as previously observed. Stresses calculated from the flexure formula are called **bending stresses** or **flexural stresses**.

[3]Moments of inertia of areas are discussed in Appendix D, Section D.3.

FIGURE 5-11

Relationships between signs of bending moments and directions of normal stresses: (a) positive bending moment and (b) negative bending moment



If the bending moment in the beam is positive, the bending stresses will be positive (tension) over the part of the cross section where y is negative, that is, over the lower part of the beam. The stresses in the upper part of the beam will be negative (compression). If the bending moment is negative, the stresses will be reversed. These relationships are shown in Fig. 5-11.

Maximum Stresses at a Cross Section

The maximum tensile and compressive bending stresses acting at any given cross section occur at points located farthest from the neutral axis. Denote by c_1 and c_2 the distances from the neutral axis to the extreme elements in the positive and negative y directions, respectively (see Fig. 5-9b and Fig. 5-11). Then the corresponding **maximum normal stresses** σ_1 and σ_2 (from the flexure formula) are

$$\sigma_1 = -\frac{Mc_1}{I} = -\frac{M}{S_1} \qquad \sigma_2 = \frac{Mc_2}{I} = \frac{M}{S_2} \tag{5-15a,b}$$

in which

$$S_1 = \frac{I}{c_1} \qquad S_2 = \frac{I}{c_2} \tag{5-16a,b}$$

The quantities S_1 and S_2 are known as the **section moduli** of the cross-sectional area. From [Eqs. (5-16a and b)], note that each section modulus has dimensions of a length to the third power (for example, in^3 or mm^3). Also note that the distances c_1 and c_2 to the top and bottom of the beam are always taken as positive quantities.

The advantage of expressing the maximum stresses in terms of section moduli arises from the fact that each section modulus combines the beam's relevant cross-sectional properties into a single quantity. Then this quantity can be listed in tables and handbooks as a property of the beam, which is a convenience to designers. (Design of beams using section moduli is explained in the next section.)

Doubly Symmetric Shapes

If the cross section of a beam is symmetric with respect to the z axis as well as the y axis (*doubly symmetric cross section*), then $c_1 = c_2 = c$, and the maximum tensile and compressive stresses are equal numerically:

$$\sigma_1 = -\sigma_2 = -\frac{Mc}{I} = -\frac{M}{S} \quad \text{or} \quad \sigma_{max} = \frac{M}{S} \tag{5-17a,b}$$

in which

$$S = \frac{I}{c} \tag{5-18}$$

is the only section modulus for the cross section.

For a beam of **rectangular cross section** with width b and height h (Fig. 5-12a), the moment of inertia and section modulus are

$$I = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6} \tag{5-19a,b}$$

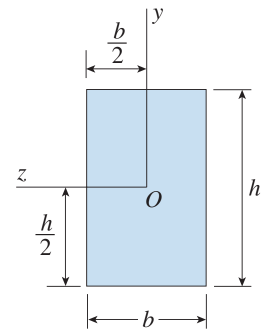
For a **circular cross section** of diameter d (Fig. 5-12b), these properties are

$$I = \frac{\pi d^4}{64} \qquad S = \frac{\pi d^3}{32} \tag{5-20a,b}$$

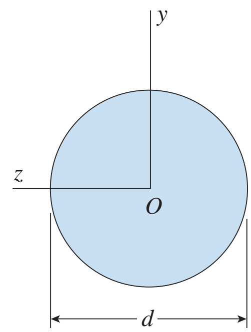
Properties of other doubly symmetric shapes, such as hollow tubes (either rectangular or circular) and wide-flange shapes, can be readily obtained from the preceding formulas.

FIGURE 5-12

Doubly symmetric cross-sectional shapes



(a)



(b)

Properties of Beam Cross Sections

Moments of inertia of many plane figures are listed in Appendix E for convenient reference. Also, the dimensions and properties of standard sizes of steel and wood beams are listed in Appendixes F and G and in many engineering handbooks, as explained in more detail in the next section.

For other cross-sectional shapes, determine the location of the neutral axis, the moment of inertia, and the section moduli by direct calculation, using the techniques described in Appendix D. This procedure is illustrated later in Example 5-4.

Limitations

The analysis presented in this section is for the pure bending of prismatic beams composed of homogeneous, linearly elastic materials. If a beam is subjected to nonuniform bending, the shear forces will produce *warping* (or out-of-plane distortion) of the cross sections. Thus, a cross section that was plane before bending is no longer plane after bending. Warping due to shear deformations greatly complicates the behavior of the beam. However, detailed investigations show that the normal stresses calculated from the flexure formula are not significantly altered by the presence of shear stresses and the associated warping (Ref. 2-1, pp. 42 and 48). Thus, you may justifiably use the theory of pure bending for calculating normal stresses in beams subjected to nonuniform bending.[4]

The flexure formula gives results that are accurate only in regions of the beam where the stress distribution is not disrupted by changes in the shape of the beam or by discontinuities in loading. For instance, the flexure formula is not applicable near the supports of a beam or close to a concentrated load. Such irregularities produce localized stresses, or *stress concentrations*, that are much greater than the stresses obtained from the flexure formula (see Section 5.13).

[4]Beam theory began with Galileo Galilei (1564–1642), who investigated the behavior of various types of beams. His work in mechanics of materials is described in his famous book *Two New Sciences*, first published in 1638 (Ref. 5-2). Although Galileo made many important discoveries regarding beams, he did not obtain the stress distribution used today. Further progress in beam theory was made by Mariotte, Jacob Bernoulli, Euler, Parent, Saint-Venant, and others (Ref. 5-3).

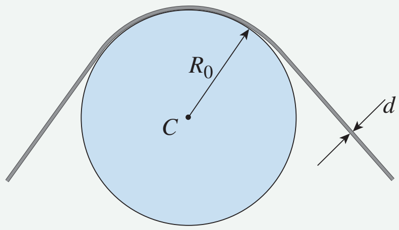
Example 5-2

A high-strength steel wire with a diameter d is bent around a cylindrical drum of radius R_0 (Fig. 5-13).

Determine the bending moment M and maximum bending stress σ_{max} in the wire, assuming $d = 4$ mm and $R_0 = 0.5$ m. (The steel wire has a modulus of elasticity $E = 200$ GPa and a proportional limit $\sigma_{pl} = 1200$ MPa.)

FIGURE 5-13

Example 5-2: Wire bent around a drum



Solution:

Use a four-step problem-solving approach.

1. ***Conceptualize*:** The first step in this example is to determine the radius of curvature ρ of the bent wire. Knowing ρ, then find the bending moment and maximum stresses.
2. ***Categorize*:**

Radius of curvature: The radius of curvature of the bent wire is the distance from the center of the drum to the neutral axis of the cross section of the wire:

$$\rho = R_0 + \frac{d}{2} \tag{5-21}$$

Bending moment: The bending moment in the wire may be found from the moment-curvature relationship (Eq. 5-13):

$$M = \frac{EI}{\rho} = \frac{2EI}{2R_0 + d} \tag{5-22}$$

in which I is the moment of inertia of the cross-sectional area of the wire. Substitute for I in terms of the diameter d of the wire [Eq. (5-20a)] to get

$$M = \frac{\pi E d^4}{32(2R_0 + d)} \tag{5-23}$$

This result was obtained without regard to the *sign* of the bending moment, since the direction of bending is obvious from the figure.

Maximum bending stresses: The maximum tensile and compressive stresses, which are equal numerically, are obtained from the flexure formula as given by Eq. (5-17b):

$$\sigma_{max} = \frac{M}{S}$$

in which S is the section modulus for a circular cross section. Substitute for M from Eq. (5-23) and for S from Eq. (5-20b) to get

$$\sigma_{max} = \frac{Ed}{2R_0 + d} \tag{5-24}$$

This same result can be obtained directly from Eq. (5-8) by replacing y with $d/2$ and substituting for ρ from Eq. (5-21).

Inspection of Fig. 5-13 reveals that the stress is compressive on the lower (or inner) part of the wire and tensile on the upper (or outer) part.

3. ***Analyze***:

Numerical results: Now substitute the given numerical data into Eqs. (5-23) and (5-24) and obtain

$$M = \frac{\pi E d^4}{32(2R_0 + d)} = \frac{\pi(200 \text{ GPa})(4 \text{ mm})^4}{32[2(0.5 \text{ m}) + 4 \text{ mm}]} = 5.01 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{Ed}{2R_0 + d} = \frac{(200 \text{ GPa})(4 \text{ mm})}{2(0.5 \text{ m}) + 4 \text{ mm}} = 797 \text{ MPa}$$

4. ***Finalize***: Maximum stress σ_{max} is less than the proportional limit of the steel wire; therefore, the calculations are valid.

Note: Because the radius of the drum is large compared to the diameter of the wire, d in comparison with $2R_0$ in the denominators of the expressions for M and σ_{max} can be safely disregarded. Then Eqs. (5-23) and (5-24) give

$$M = 5.03 \text{ N} \cdot \text{m} \qquad \sigma_{max} = 800 \text{ MPa}$$

These results are on the conservative side and differ by less than 1% from the more precise values.

Example 5-3

A simple beam with an overhang (from Examples 4-5 and 4-9) is shown in Fig. 5-14. A uniform load with an intensity $q = 400$ lb/ft acts throughout the length of the beam, and a concentrated load $P = 2400$ lb acts at a point 9 ft from the left-hand support. Uniform load q includes the weight of the beam. The beam is constructed of structural glued and laminated timber, has a cross section width of $b = 5$ in., and has a height of $h = 22$ in. (Fig. 5-15).

(a) Determine the maximum tensile and compressive stresses in the beam due to bending.

(b) If load q is unchanged, find the maximum permissible value of load P if the allowable normal stress in tension and compression is $\sigma_a = 1875$ psi.

FIGURE 5-14

Beam with an overhang and uniform and concentrated loads

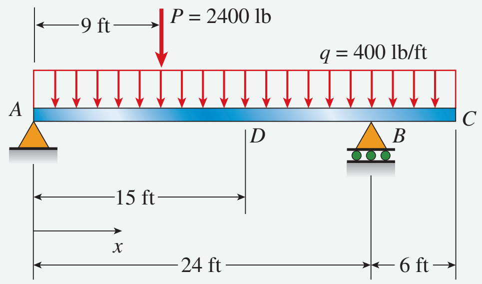
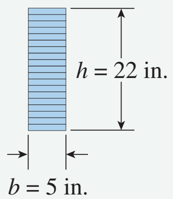


FIGURE 5-15

Beam cross section



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

Part (a): Maximum normal stresses.

1, 2. ***Conceptualize, Categorize*:** Begin the analysis by drawing the shear-force and bending-moment diagrams (Fig. 5-16); then determine the maximum bending moment, which occurs under the concentrated load. This is detailed in Example 4-9, and the resulting diagrams are shown in Fig. 5-16. The moment diagram shows that $M_{max} = 37{,}800$ lb-ft at 9 ft to the right of support A. The maximum bending stresses in the beam occur at the cross section of the maximum moment.

Section modulus: The section modulus for the rectangular cross-sectional area in Fig. 5-15 is from Eq. (5-19b):

$$S = \frac{bh^2}{6} = \frac{1}{6}(5\text{ in.})(22\text{ in.})^2 = 403.3\text{ in}^3 \qquad \textbf{(a)}$$

3. ***Analyze*:**

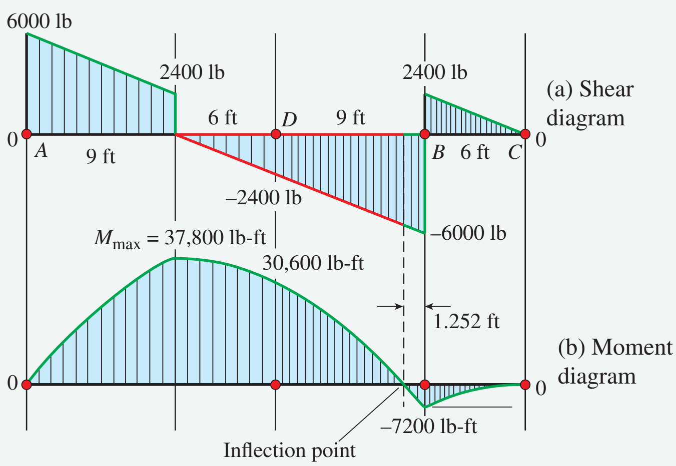
Maximum stresses: The maximum tensile and compressive stresses are obtained from Eq. (5-17):

$$\sigma_t = \frac{M_{max}}{S} = \frac{(37{,}800\text{ lb-ft})(12\text{ in./ft})}{403.3\text{ in}^3} = 1125\text{ psi}$$

$$\sigma_c = -\frac{M_{max}}{S} = -1125\text{ psi} \qquad \textbf{(b)}$$

FIGURE 5-16

(a, b) Shear and moment diagrams (from Example 4-9)



4. ***Finalize***: The moment diagram is plotted on the compression side of the beam, so most of span AB has compressive stress on the top and tension stress on the bottom of the beam. The reverse is true for the portion of the beam to the right of the inflection point, which includes overhang segment BC.

Part (b): Maximum permissible load *P*.

1, 2. ***Conceptualize, Categorize***: The normal stresses in Eq. (b) at the location of the maximum moment are well below the allowable value of 1875 psi, so the beam can carry a much larger value of load P than that applied in part (a). Let the distance from support A to load P be $a = 9$ ft, span AB length $L = 24$ ft, and the uniform load be unchanged at $q = 400$ lb/ft.

3. ***Analyze***: Apply concentrated load P and uniform load q and solve for the reaction at A:

$$R_A = P\left(\frac{L-a}{L}\right) + \frac{15}{32}qL \tag{c}$$

The maximum moment is at distance a from support A and is written as

$$M_{\max} = R_A a - \frac{qa^2}{2} \tag{d}$$

Equate $M_{\max}$ to $(\sigma_a)(S) = 63{,}016$ lb-ft, insert numerical values in Eqs. (c) and (d), and solve for $P_{\max} = 6883$ lb.

Alternate solution: Apply additional load ΔP to increase the maximum moment from 37,800 lb-ft to 63,016 lb-ft, that is, $\Delta M = 25{,}216$ lb-ft. The required additional load ΔP is computed using Eq. (4-13), which gives the moment at the location of a concentrated load:

$$\Delta P = \frac{L}{a(L-a)}\Delta M = \frac{24\text{ ft}}{9\text{ ft}(24\text{ ft} - 9\text{ ft})}(25{,}216\text{ lb-ft}) = 4483\text{ lb} \tag{e}$$

Add ΔP to the load $P = 2400$ lb from part (a) to get

$$P_{\max} = P + \Delta P = 2400\text{ lb} + 4483\text{ lb} = 6883\text{ lb} \tag{f}$$

4. ***Finalize***: Check that the maximum permissible value of P produces normal stresses at the allowable level at the point of maximum moment. Substitute $P_{\max}$ into Eqs. (c) and (d) to find that $R_A = 8802$ lb and $M_{\max} = 63{,}016$ lb-ft. Using these values, the stresses at the point of load $P_{\max}$ application are

$$\sigma_t = -\sigma_c = \frac{M_{\max}}{S} = \frac{(63{,}016\text{ lb-ft})(12\text{ in/ft})}{403.3\text{ in}^3} = 1875\text{ psi} \tag{g}$$

Example 5-4

The beam ABC shown in Fig. 5-17a has simple supports at A and B and an overhang from B to C. The length of the span is $L = 3.0$ m, and the length of the overhang is $L/2 = 1.5$ m. A uniform load of intensity $q = 3.2$ kN/m acts throughout the entire length of the beam (4.5 m).

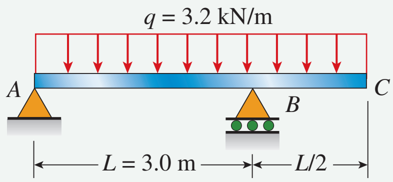
The beam has a cross section of channel shape with a width of $b = 300$ mm and height of $h = 80$ mm (Fig. 5-18). The web thickness is $t = 12$ mm, and the average thickness of the sloping flanges is the same. For the purpose of calculating the properties of the cross section, assume that the cross section consists of three rectangles, as shown in Fig. 5-18b.

(a) Determine the maximum tensile and compressive stresses in the beam due to the uniform load.

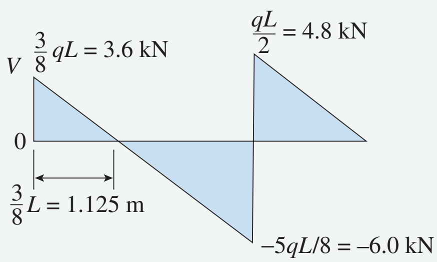
(b) Find the maximum permissible value of uniform load q (in kN/m) if allowable stresses in tension and compression are $\sigma_{aT} = 110$ MPa and $\sigma_{aC} = 92$ MPa, respectively.

FIGURE 5-17

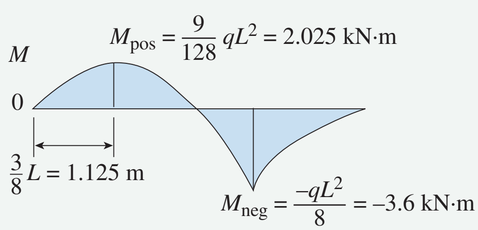
Example 5-4: Stresses in a beam with an overhang



(a) Beam with an overhang



(b) Shear diagram



(c) Moment diagram

Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

Part (a): Maximum tensile and compressive stresses.

1, 2. ***Conceptualize, Categorize***: Reactions, shear forces, and bending moments are computed in the analysis of this beam. First, find the reactions at supports A and B using statics, as described in Chapter 4. The results are

$$R_A = \frac{3}{8}qL = 3.6 \text{ kN} \qquad R_B = \frac{9}{8}qL = 10.8 \text{ kN}$$

From these values, construct the shear-force diagram (Fig. 5-17b). Note that the shear force changes sign and is equal to zero at two locations: (1) at a distance of 1.125 m from the left-hand support and (2) at the right-hand reaction.

Next, draw the bending-moment diagram shown in Fig. 5-17c. Both the maximum positive and maximum negative bending moments occur at the cross sections where the shear force changes sign. These maximum moments are

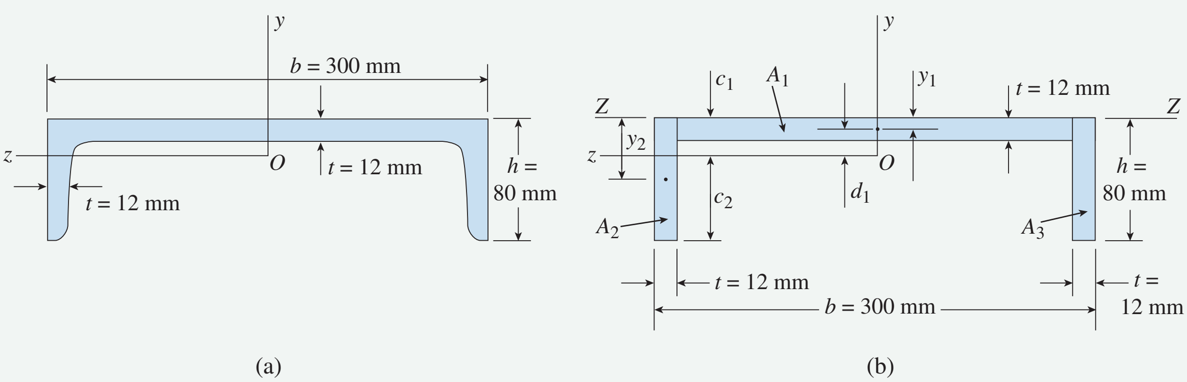
$$M_{\text{pos}} = \frac{9}{128}qL^2 = 2.025 \text{ kN} \cdot \text{m} \qquad M_{\text{neg}} = \frac{-qL^2}{8} = -3.6 \text{ kN} \cdot \text{m}$$

respectively.

Neutral axis of the cross section (Fig. 5-18b): The origin O of the y-z coordinates is placed at the centroid of the cross-sectional area; therefore, the z axis becomes the neutral axis of the cross section. The centroid is located by using the techniques described in Appendix D, Section D.2, as follows.

FIGURE 5-18

Cross section of beam discussed in Example 5-4: (a) actual shape and (b) idealized shape for use in analysis (the thickness of the beam is exaggerated for clarity)



First, divide the area into three rectangles (A_1, A_2, and A_3). Second, establish a reference axis Z–Z across the upper edge of the cross section, and let y_1 and y_2 be the distances from the Z–Z axis to the centroids of areas A_1 and A_2, respectively. Then the calculations for locating the centroid of the entire channel section (distances c_1 and c_2) are

Area 1: $y_1 = t/2 = 6 \text{ mm}$

$$A_1 = (b - 2t)(t) = (276 \text{ mm})(12 \text{ mm}) = 3312 \text{ mm}^2$$

Area 2: $y_2 = h/2 = 40 \text{ mm}$

$$A_2 = ht = (80 \text{ mm})(12 \text{ mm}) = 960 \text{ mm}^2$$

Area 3: $y_3 = y_2 \quad A_3 = A_2$

$$c_1 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{y_1 A_1 + 2y_2 A_2}{A_1 + 2A_2}$$

$$= \frac{(6 \text{ mm})(3312 \text{ mm}^2) + 2(40 \text{ mm})(960 \text{ mm}^2)}{3312 \text{ mm}^2 + 2(960 \text{ mm}^2)} = 18.48 \text{ mm}$$

$$c_2 = h - c_1 = 80 \text{ mm} - 18.48 \text{ mm} = 61.52 \text{ mm}$$

Thus, the position of the neutral axis (the z axis) is determined.

Moment of inertia: In order to calculate the stresses from the flexure formula, determine the moment of inertia of the cross-sectional area with respect to the neutral axis. These calculations require the use of the parallel axis theorem (see Appendix D, Section D.4).

Beginning with area A_1, obtain its moment of inertia $(I_z)_1$ about the z axis from the equation

$$(I_z)_1 = (I_c)_1 + A_1 d_1^2 \qquad \textbf{(a)}$$

In this equation, $(I_c)_1$ is the moment of inertia of area A_1 about its own centroidal axis:

$$(I_c)_1 = \frac{1}{12}(b - 2t)(t)^3 = \frac{1}{12}(276 \text{ mm})(12 \text{ mm})^3 = 39{,}744 \text{ mm}^4$$

and d_1 is the distance from the centroidal axis of area A_1 to the z axis:

$$d_1 = c_1 - t/2 = 18.48 \text{ mm} - 6 \text{ mm} = 12.48 \text{ mm}$$

Therefore, the moment of inertia of area A_1 about the z axis [from Eq. (a)] is

$$(I_z)_1 = 39{,}744 \text{ mm}^4 + (3312 \text{ mm}^2)(12.48 \text{ mm})^2 = 555{,}600 \text{ mm}^4$$

Proceed in the same manner for areas A_2 and A_3 to get

$$(I_z)_2 = (I_z)_3 = 956{,}600 \text{ mm}^4$$

Thus, the centroidal moment of inertia I_z of the entire cross-sectional area is

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 = 2.469 \times 10^6 \text{ mm}^4$$

Section moduli: The section moduli for the top and bottom of the beam, respectively, are

$$S_1 = \frac{I_z}{c_1} = 133{,}600 \text{ mm}^3 \qquad S_2 = \frac{I_z}{c_2} = 40{,}100 \text{ mm}^3$$

[see Eqs. (5-16a and b)]. With the cross-sectional properties determined, now calculate the maximum stresses from Eqs. (5-15a and b).

3. ***Analyze***:

Maximum stresses: At the cross section of maximum positive bending moment, the largest tensile stress occurs at the bottom of the beam (σ_2) and the largest compressive stress occurs at the top (σ_1). Thus, from Eqs. (5-15b) and (5-15a), respectively, you get

$$\sigma_t = \sigma_2 = \frac{M_{pos}}{S_2} = \frac{2.025 \text{ kN} \cdot \text{m}}{40{,}100 \text{ mm}^3} = 50.5 \text{ MPa}$$

$$\sigma_c = \sigma_1 = -\frac{M_{pos}}{S_1} = -\frac{2.025 \text{ kN} \cdot \text{m}}{133{,}600 \text{ mm}^3} = -15.2 \text{ MPa}$$

Similarly, the largest stresses at the section of maximum negative moment are

$$\sigma_t = \sigma_1 = -\frac{M_{neg}}{S_1} = -\frac{-3.6 \text{ kN} \cdot \text{m}}{133{,}600 \text{ mm}^3} = 26.9 \text{ MPa}$$

$$\sigma_c = \sigma_2 = \frac{M_{neg}}{S_2} = \frac{-3.6 \text{ kN} \cdot \text{m}}{40{,}100 \text{ mm}^3} = -89.8 \text{ MPa}$$

4. ***Finalize***: A comparison of these four stresses shows that the largest tensile stress in the beam is 50.5 MPa and occurs at the bottom of the beam at the cross section of maximum positive bending moment; thus,

$$(\sigma_t)_{max} = 50.5 \text{ MPa}$$

The largest compressive stress is −89.8 MPa and occurs at the bottom of the beam at the section of maximum negative moment:

$$(\sigma_c)_{max} = -89.8 \text{ MPa}$$

Recall that these are the maximum bending stresses due to the uniform load acting on the beam.

Part (b): Maximum permissible value of uniform load q.

1, 2. ***Conceptualize, Categorize***: Next, find q_{max} based on the given allowable normal stresses, which are different for tension and compression. The allowable compression stress is σ_{aC} lower than that for tension, σ_{aT}, to account for the possibility of local buckling of the flanges of the C shape (if they are in compression).

Use the flexure formula to compute potential values of q_{max} at four locations: at the top and bottom of the beam at the location of the maximum positive moment (M_{pos}) and at the top and bottom of the beam at the location of the maximum negative moment (M_{neg}). In each case, be sure to use the proper value of allowable stress. Assume that the C shape is used in the orientation shown in Fig. 5-18 (flanges downward), so at the location of M_{pos}, the top of the beam is in compression and the bottom is in tension, while the opposite is true at point B. Using the expressions for M_{pos} and M_{neg} and equating each to the appropriate product of allowable stress and section modulus, solve for possible values of q_{max} as given here.

3. ***Analyze***: In beam segment AB at the *top* of beam,

$$M_{\text{pos}} = \frac{9}{128}q_1L^2 = \sigma_{aC}S_1 \qquad \text{so} \qquad q_1 = \frac{128}{9L^2}(\sigma_{aC}S_1) = 19.42 \text{ kN/m}$$

In beam segment AB at the *bottom* of beam,

$$M_{\text{pos}} = \frac{9}{128}q_2L^2 = \sigma_{aT}S_2 \qquad \text{so} \qquad q_2 = \frac{128}{9L^2}(\sigma_{aT}S_2) = 6.97 \text{ kN/m}$$

At joint B at the top of beam,

$$M_{\text{pos}} = \frac{1}{8}q_3L^2 = \sigma_{aT}S_1 \qquad \text{so} \qquad q_3 = \frac{8}{L^2}(\sigma_{aT}S_1) = 13.06 \text{ kN/m}$$

At joint B at *bottom* of the beam,

$$M_{\text{pos}} = \frac{1}{8}q_4L^2 = \sigma_{aC}S_2 \qquad \text{so} \qquad q_4 = \frac{8}{L^2}(\sigma_{aC}S_2) = 3.28 \text{ kN/m}$$

4. ***Finalize***: From these calculations, the bottom of the beam near joint B (where the flange tips are in compression) does indeed control the maximum permissible value of uniform load q. Hence,

$$q_{\max} = 3.28 \text{ kN/m}$$

5.6 Design of Beams for Bending Stresses

The process of designing a beam requires that many factors be considered, including the type of structure (airplane, automobile, bridge, building, or whatever), the materials to be used, the loads to be supported, the environmental conditions to be encountered, and the costs to be paid. However, from the standpoint of strength, the task eventually reduces to selecting a shape and size of beam such that the actual stresses in the beam do not exceed the allowable stresses for the material. This section considers only the bending stresses [that is, the stresses obtained from the flexure formula, Eq. (5-14)].

When designing a beam to resist bending stresses, begin by calculating the **required section modulus**. For instance, if the beam has a doubly symmetric cross section and the allowable stresses are the same for both tension and compression, calculate the required modulus by dividing the maximum bending moment by the allowable bending stress for the material [see Eq. (5-17)]:

$$S = \frac{M_{\max}}{\sigma_{\text{allow}}} \tag{5-25}$$

The allowable stress is based upon the properties of the material and the desired factor of safety. To ensure that this stress is not exceeded, choose a beam that provides a section modulus at least as large as that obtained from Eq. (5-25).

If the cross section is not doubly symmetric, or if the allowable stresses are different for tension and compression, it may be necessary to determine two required section moduli—one based upon tension and the other based upon compression. Then provide a beam that satisfies both criteria.

To minimize weight and save material, select a beam that has the least cross-sectional area while still providing the required section moduli (and also meeting any other design requirements that may be imposed).

Beams are constructed in a great variety of shapes and sizes to suit a myriad of purposes. For instance, very large steel beams are fabricated by welding (Fig. 5-19), aluminum beams are extruded as round or rectangular tubes, wood beams are cut and glued to fit special requirements, and reinforced concrete beams are cast in any desired shape by proper construction of the forms.

In addition, beams of steel, aluminum, plastic, and wood can be ordered in **standard shapes and sizes** from catalogs supplied by dealers and manufacturers. Readily available shapes include wide-flange beams, I-beams, angles, channels, rectangular beams, and tubes.

FIGURE 5-19

Welder fabricating a large wide flange steel beam

(Courtesy of AISC)

Beams of Standardized Shapes and Sizes

The dimensions and properties of many kinds of beams are listed in engineering handbooks. For instance, in the United States, the shapes and sizes of structural-steel beams are standardized by the American Institute of Steel Construction (AISC), which publishes manuals giving their properties in both USCS and SI units (Ref. 5-4). The tables in these manuals list cross-sectional dimensions and properties such as weight, cross-sectional area, moment of inertia, and section modulus.

Properties of aluminum and wood beams are tabulated in a similar manner and are available in publications of the Aluminum Association (Ref. 5-5) and the American Forest and Paper Association (Ref. 5-6).

Abridged tables of steel beams and wood beams are given later in this book for use in solving problems using both USCS and SI units (see Appendixes F and G).

Structural-steel sections are given a designation such as W 30 × 211 in USCS units, which means that the section is of W shape (also called a wide-flange shape) with a nominal depth of 30 in. and a weight of 211 lb per ft of length (see Table F-1(a), Appendix F). The corresponding properties for each W shape are also given in SI units in Table F-1(b). For example, in SI units, the W 30 × 211 is listed as W 760 × 314 with a nominal depth of 760 millimeters and mass of 314 kilograms per meter of length.

Similar designations are used for S shapes (also called I-beams) and C shapes (also called channels), as shown in Tables F-2(a) and F-3(a) in USCS units and in Tables F-2(b) and F-3(b) in SI units. Angle sections, or L shapes, are designated by the lengths of the two legs and the thickness (see Tables F-4 and F-5). For example, L 8 × 6 × 1 [see Table F-5(a)] denotes an angle with unequal legs, one of length 8 in. and the other of length 6 in., with a thickness of 1 in. The corresponding label in SI units for this unequal leg angle is L 203 × 152 × 25.4 [see Table F-5(b)].

The standardized steel sections described here are manufactured by *rolling*, a process in which a billet of hot steel is passed back and forth between rolls until it is formed into the desired shape.

Aluminum structural sections are usually made by the process of *extrusion*, in which a hot billet is pushed, or extruded, through a shaped die. Since dies are relatively easy to make and the material is workable; aluminum beams can be extruded in almost any desired shape. Standard shapes of wide-flange beams, I-beams, channels, angles, tubes, and other sections are listed in the *Aluminum Design Manual* (Ref. 5-5). In addition, custom-made shapes can be ordered.

Most **wood beams** have rectangular cross sections and are designated by nominal dimensions, such as 4 × 8 inches. These dimensions represent the rough-cut size of the lumber. The net dimensions (or actual dimensions) of a wood beam are smaller than the nominal dimensions if the sides of the rough lumber have been planed, or *surfaced*, to make them smooth. Thus, a 4 × 8 wood beam has actual dimensions of 3.5 × 7.25 in. after it has been surfaced. Of course, the net dimensions of surfaced lumber should be used in all engineering computations. Therefore, net dimensions and the corresponding properties (in USCS units) are given in Appendix G. Similar tables are available in SI units.

Relative Efficiency of Various Beam Shapes

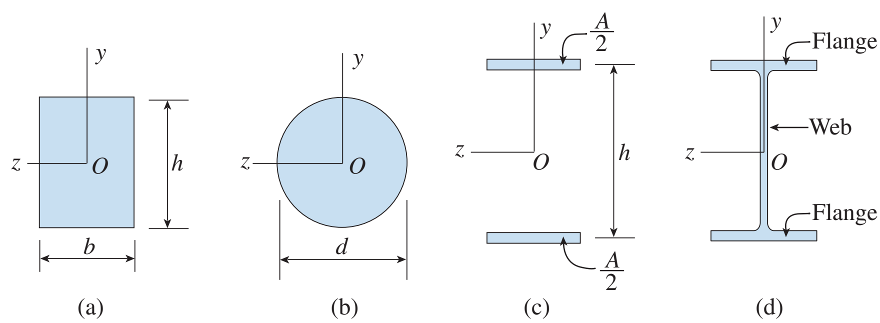
One of the objectives in designing a beam is to use the material as efficiently as possible within the constraints imposed by function, appearance, manufacturing costs, and the like. From the standpoint of strength alone, efficiency in bending depends primarily upon the shape of the cross section. In particular, the most efficient beam is one in which the material is located as far as practical from the neutral axis. The farther a given amount of material is from the neutral axis, the larger the section modulus becomes—and the larger the section modulus, the larger the bending moment that can be resisted (for a given allowable stress).

As an illustration, consider a cross section in the form of a **rectangle** of width b and height h (Fig. 5-20a). The section modulus [from Eq. (5-19b)] is

$$S = \frac{bh^2}{6} = \frac{Ah}{6} = 0.167Ah \tag{5-26}$$

where A denotes the cross-sectional area. This equation shows that a rectangular cross section of given area becomes more efficient as the height h is increased (and the width b is decreased to keep the area constant). Of course, there is a practical limit to the increase in height, because the beam becomes laterally unstable when the ratio of height to width becomes too large. Thus, a beam of very narrow rectangular section will fail due to lateral (sideways) buckling rather than to insufficient strength of the material.

FIGURE 5-20
Cross-sectional shapes of beams



Next, compare a **solid circular cross section** of diameter d (Fig. 5-20b) with a square cross section of the same area. The side h of a square having the same area as the circle is $h = (d/2)\sqrt{\pi}$. The corresponding section moduli [from Eqs. (5-19b) and (5-20b)] are

$$S_{\text{square}} = \frac{h^3}{6} = \frac{\pi\sqrt{\pi}d^3}{48} = 0.1160d^3 \tag{5-27a}$$

$$S_{\text{circle}} = \frac{\pi d^3}{32} = 0.0982d^3 \tag{5-27b}$$

which gives

$$\frac{S_{\text{square}}}{S_{\text{circle}}} = 1.18 \tag{5-28}$$

This result shows that a beam of square cross section is more efficient in resisting bending than is a circular beam of the same area. The reason, of course, is that a circle has a relatively larger amount of material located near the neutral axis. This material is less highly stressed; therefore, it does not contribute as much to the strength of the beam.

The **ideal cross-sectional shape** for a beam of given cross-sectional area A and height h would be obtained by placing one-half of the area at a distance $h/2$ above the neutral axis and the other half at distance $h/2$ below the neutral axis, as shown in Fig. 5-20c. For this ideal shape, obtain

$$I = 2\left(\frac{A}{2}\right)\left(\frac{h}{2}\right)^2 = \frac{Ah^2}{4} \qquad S = \frac{I}{h/2} = 0.5Ah \tag{5-29a,b}$$

These theoretical limits are approached in practice by wide-flange sections and I-sections, which have most of their material in the flanges (Fig. 5-20d). For standard wide-flange beams, the section modulus is approximately

$$S \approx 0.35Ah \tag{5-30}$$

which is less than the ideal but much larger than the section modulus for a rectangular cross section of the same area and height [see Eq. (5-26)].

Another desirable feature of a wide-flange beam is its greater width; hence, its greater stability with respect to sideways buckling when compared to a rectangular beam of the same height and section modulus. On the other hand, there are practical limits to how thin the web can be for a wide-flange beam. A web that is too thin is susceptible to localized buckling or it may be overstressed in shear (see Section 5.10).

The following four examples illustrate the process of selecting a beam on the basis of the allowable stresses. Only the effects of bending stresses (obtained from the flexure formula) are considered.

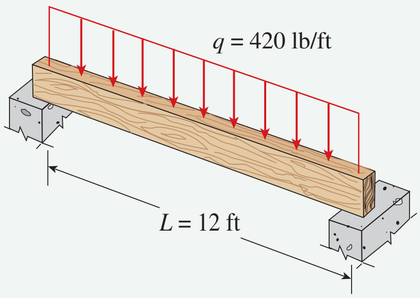
Note: When solving examples and problems that require the selection of a steel or wood beam from the tables in the appendixes, use the following rule: *If several choices are available in a table, select the lightest beam that provides the required section modulus.*

Example 5-5

A simply supported wood beam with a span length $L = 12$ ft carries a uniform load $q = 420$ lb/ft (Fig. 5-21). The allowable bending stress is 1800 psi, the wood weighs 35 lb/ft^3, and the beam is supported laterally against sideways buckling and tipping.

Select a suitable size for the beam from the table in Appendix G.

FIGURE 5-21
Example 5-5: Design of a simply supported wood beam



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. ***Conceptualize, Categorize***: Since the beam weight is not known in advance, proceed by trial-and-error:

i. Calculate the required section modulus based upon the given uniform load.

ii. Select a trial size for the beam.

iii. Add the weight of the beam to the uniform load and calculate a new required section modulus.

Check to see that the selected beam is still satisfactory. If it is not, select a larger beam and repeat the process.

3. ***Analyze***:

i. The maximum bending moment in the beam occurs at the midpoint:

$$M_{\max} = \frac{qL^2}{8} = \frac{(420 \text{ lb/ft})(12 \text{ ft})^2(12 \text{ in./ft})}{8} = 90{,}720 \text{ lb-in.}$$

The required section modulus [Eq. (5-25)] is

$$S = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{90{,}720 \text{ lb-in.}}{1800 \text{ psi}} = 50.40 \text{ in}^3$$

ii. From the table in Appendix G, the lightest beam that supplies a section modulus of at least 50.40 in^3 about axis 1–1 is a 3×12 in. beam (nominal dimensions). This beam has a section modulus equal to 52.73 in^3 and weighs 6.8 lb/ft. (Note that Appendix G gives weights of beams based upon a density of 35 lb/ft^3.)

iii. The uniform load on the beam now becomes 426.8 lb/ft, and the corresponding required section modulus is

$$S = (50.40 \text{ in}^3)\left(\frac{426.8 \text{ lb/ft}}{420 \text{ lb/ft}}\right) = 51.22 \text{ in}^3$$

4. ***Finalize***: The previously selected beam has a section modulus of 52.73 in^3, which is larger than the required modulus of 51.22 in^3.

Therefore, a 3 × 12 in. beam is satisfactory.

Note: If the weight density of the wood is other than 35 lb/ft^3, compute the weight of the beam per linear foot by multiplying the value in the last column in Appendix G by the ratio of the actual weight density to 35 lb/ft^3.

Example 5-6

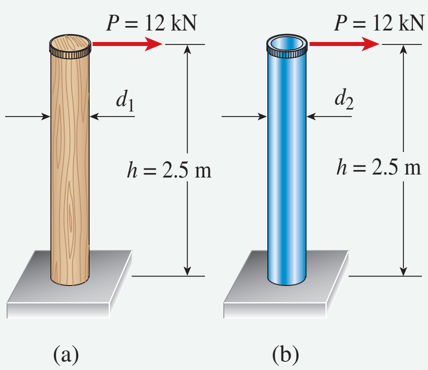
A vertical post 2.5-meters high must support a lateral load $P = 12$ kN at its upper end (Fig. 5-22). Two plans are proposed—a solid wood post and a hollow aluminum tube.

(a) What is the minimum required diameter d_1 of the wood post if the allowable bending stress in the wood is 15 MPa?

(b) What is the minimum required outer diameter d_2 of the aluminum tube if its wall thickness is to be one-eighth of the outer diameter and the allowable bending stress in the aluminum is 50 MPa?

FIGURE 5-22

Example 5-6: (a) Solid wood post and (b) aluminum tube



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1. ***Conceptualize***:

Maximum bending moment: The maximum moment occurs at the base of the post and is equal to the load P times the height h; thus,

$$M_{max} = Ph = (12 \text{ kN})(2.5 \text{ m}) = 30 \text{ kN} \cdot \text{m}$$

Part (a): Wood post.

2, 3. ***Categorize, Analyze***: The required section modulus S_1 for the wood post [see Eqs. (5-20b and 5-25)] is

$$S_1 = \frac{\pi d_1^3}{32} = \frac{M_{max}}{\sigma_{allow}} = \frac{30 \text{ kN} \cdot \text{m}}{15 \text{ MPa}} = 0.0020 \text{ m}^3 = 2 \times 10^6 \text{ mm}^3$$

Solving for the diameter gives

$$d_1 = 273 \text{ mm}$$

4. ***Finalize***: The diameter selected for the wood post must be equal to or larger than 273 mm if the allowable stress is not to be exceeded.

Part (a): Aluminum tube.

2, 3. ***Categorize, Analyze***: To determine the section modulus S_2 for the tube, first find the moment of inertia I_2 of the cross section. The wall thickness of the tube is $d_2/8$; therefore, the inner diameter is $d_2 - d_2/4$, or $0.75d_2$. Thus, the moment of inertia [see Eq. (5-20a)] is

$$I_2 = \frac{\pi}{64}[d_2^4 - (0.75d_2)^4] = 0.03356d_2^4$$

The section modulus of the tube is now obtained from Eq. (5-18) as

$$S_2 = \frac{I_2}{c} = \frac{0.03356d_2^4}{d_2/2} = 0.06712d_2^3$$

The required section modulus is obtained from Eq. (5-25):

$$S_2 = \frac{M_{max}}{\sigma_{allow}} = \frac{30 \text{ kN} \cdot \text{m}}{50 \text{ MPa}} = 0.0006 \text{ m}^3 = 600 \times 10^3 \text{ mm}^3$$

Equate the two preceding expressions for the section modulus, then solve for the required outer diameter:

$$d_2 = \left(\frac{600 \times 10^3 \text{ mm}^3}{0.06712}\right)^{1/3} = 208 \text{ mm}$$

4. ***Finalize***: The corresponding inner diameter is 0.75(208 mm), or 156 mm.

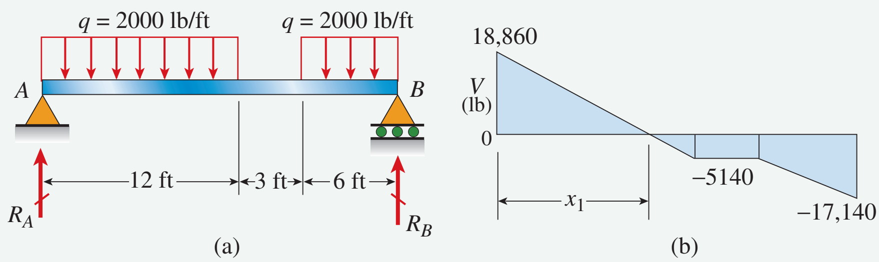
Example 5-7

A simple beam AB of span length 21 ft must support a uniform load $q = 2000$ lb/ft distributed along the beam in the manner shown in Fig. 5-23a.

Considering both the uniform load and the weight of the beam, and also using an allowable bending stress of 18,000 psi, select a structural steel beam of wide-flange shape to support the loads.

FIGURE 5-23

Example 5-7: Design of a simple beam with partial uniform loads



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. ***Conceptualize, Categorize*:** In this example, proceed as follows:

- **i.** Find the maximum bending moment in the beam due to the uniform load.
- **ii.** Knowing the maximum moment, find the required section modulus.
- **iii.** Select a trial wide-flange beam from Table F-1 in Appendix F and obtain the weight of the beam.
- **iv.** With the weight known, calculate a new value of the bending moment and a new value of the section modulus.

Determine whether the selected beam is still satisfactory. If it is not, select a new beam size and repeat the process until a satisfactory size of beam has been found.

Maximum bending moment: To assist in locating the cross section of maximum bending moment, construct the shear-force diagram (Fig. 5-23b) using the methods described in Chapter 4. As part of that process, determine the reactions at the supports:

$$R_A = 18{,}860 \text{ lb} \qquad R_B = 17{,}140 \text{ lb}$$

The distance x_1 from the left-hand support to the cross section of zero shear force is obtained from

$$V = R_A - qx_1 = 0$$

which is valid in the range $0 \le x \le 12$ ft. Solve for x_1 to get

$$x_1 = \frac{R_A}{q} = \frac{18{,}860 \text{ lb}}{2000 \text{ lb/ft}} = 9.430 \text{ ft}$$

which is less than 12 ft; therefore, the calculation is valid.

The maximum bending moment occurs at the cross section where the shear force is zero; therefore,

$$M_{\text{max}} = R_A x_1 - \frac{q x_1^2}{2} = 88{,}920 \text{ lb-ft}$$

3. ***Analyze***:

Required section modulus: The required section modulus (based only upon the load q) is obtained from Eq. (5-25):

$$S = \frac{M_{\text{max}}}{\sigma_{\text{allow}}} = \frac{(88{,}920 \text{ lb-ft})(12 \text{ in./ft})}{18{,}000 \text{ psi}} = 59.3 \text{ in}^3$$

Trial beam: Now turn to Table F-1 and select the lightest wide-flange beam having a section modulus greater than 59.3 in^3. The lightest beam that provides this section modulus is W 12 × 50 with $S = 64.7$ in^3. This beam weighs 50 lb/ft (Recall that the tables in Appendix F are abridged, so a lighter beam may actually be available.)

Now recalculate the reactions, maximum bending moment, and required section modulus with the beam loaded by both the uniform load q and its own weight. Under these combined loads the reactions are

$$R_A = 19{,}380 \text{ lb} \qquad R_B = 17{,}670 \text{ lb}$$

and the distance to the cross section of zero shear becomes

$$x_1 = \frac{19{,}380 \text{ lb}}{2050 \text{ lb/ft}} = 9.454 \text{ ft}$$

The maximum bending moment increases to 91,610 lb-ft, and the new required section modulus is

$$S = \frac{M_{\text{max}}}{\sigma_{\text{allow}}} = \frac{(91{,}610 \text{ lb-ft})(12 \text{ in./ft})}{18{,}000 \text{ psi}} = 61.1 \text{ in}^3$$

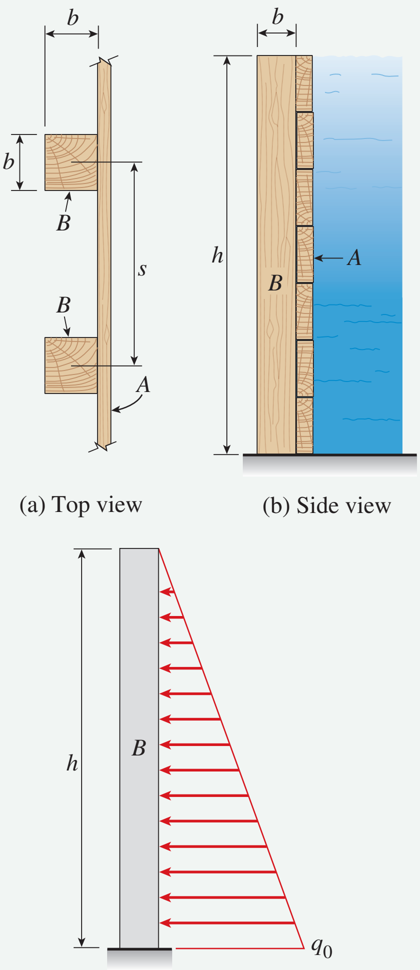
4. ***Finalize***: Thus, the W 12 × 50 beam with section modulus $S = 64.7$ in^3 is still satisfactory.

Note: If the new required section modulus exceeded that of the W 12 × 50 beam, a new beam with a larger section modulus would be selected and the process repeated.

Example 5-8

FIGURE 5-24

Example 5-8: Wood dam with horizontal planks A supported by vertical posts B



(a) Top view

(b) Side view

(c) Loading diagram

A temporary wood dam is constructed of horizontal planks A supported by vertical wood posts B that are sunk into the ground so that they act as cantilever beams (Fig. 5-24). The posts are of square cross section (dimensions $b \times b$) and spaced at distance $s = 0.8$ m, center to center. Assume that the water level behind the dam is at its full height $h = 2.0$ m.

Determine the minimum required dimension b of the posts if the allowable bending stress in the wood is $\sigma_{allow} = 8.0$ MPa.

Solution:

Use a four-step problem-solving approach.

1. ***Conceptualize***:

   Loading diagram: Each post is subjected to a triangularly distributed load produced by the water pressure acting against the planks. Consequently, the loading diagram for each post is triangular (Fig. 5-24c). The maximum intensity q_0 of the load on the posts is equal to the water pressure at depth h times the spacing s of the posts:

$$q_0 = \gamma h s \tag{a}$$

   in which γ is the specific weight of water. Note that q_0 has units of force per unit distance, γ has units of force per unit volume, and both h and s have units of length.

2. ***Categorize***:

   Section modulus: Since each post is a cantilever beam, the maximum bending moment occurs at the base and is given by

$$M_{max} = \frac{q_0 h}{2}\left(\frac{h}{3}\right) = \frac{\gamma h^3 s}{6} \tag{b}$$

   Therefore, the required section modulus [Eq. (5-25)] is

$$S = \frac{M_{max}}{\sigma_{allow}} = \frac{\gamma h^3 s}{6\sigma_{allow}} \tag{c}$$

3. ***Analyze***: For a beam of square cross section, the section modulus is $S = b^3/6$ [see Eq. (5-19b)]. Substitute this expression for S into Eq. (c) to get a formula for the cube of the minimum dimension b of the posts:

$$b^3 = \frac{\gamma h^3 s}{\sigma_{allow}} \tag{d}$$

Numerical values: Now substitute numerical values into Eq. (d) and obtain

$$b^3 = \frac{(9.81\ \text{kN/m}^3)(2.0\ \text{m})^3(0.8\ \text{m})}{8.0\ \text{MPa}} = 0.007848\ \text{m}^3 = 7.848 \times 10^6\ \text{mm}^3$$

from which

$$b = 199\ \text{mm}$$

4. ***Finalize***: Thus, the minimum required dimension b of the posts is 199 mm. Any larger dimension, such as 200 mm, ensures that the actual bending stress is less than the allowable stress.

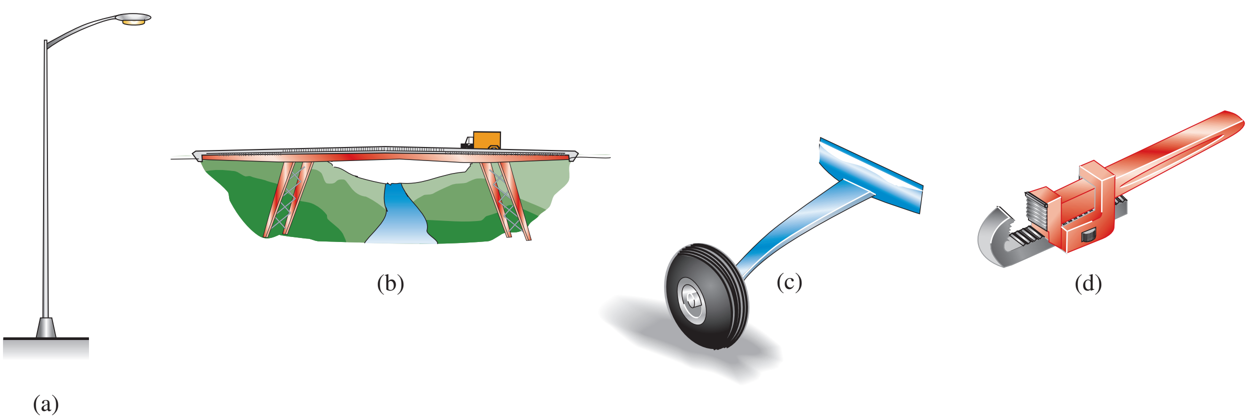
5.7 Nonprismatic Beams

The beam theories described in this chapter were derived for prismatic beams, that is, straight beams having the same cross sections throughout their lengths. However, nonprismatic beams are commonly used to reduce weight and improve appearance. Such beams are found in automobiles, airplanes, machinery, bridges, buildings, tools, and many other applications (Fig. 5-25). Fortunately, the flexure formula [Eq. (5-13)] gives reasonably accurate values for the bending stresses in nonprismatic beams whenever the changes in cross-sectional dimensions are gradual, as in the examples shown in Fig. 5-25.

The manner in which the bending stresses vary along the axis of a nonprismatic beam is not the same as for a prismatic beam. In a prismatic beam, the section modulus S is constant, so the stresses vary in direct proportion to the bending moment (because $\sigma = M/S$). However, in a nonprismatic beam, the section modulus also varies along the axis. Consequently, do not assume that the maximum stresses occur at the cross section with the largest bending moment—sometimes the maximum stresses occur elsewhere, as illustrated in Example 5-9.

FIGURE 5-25

Examples of nonprismatic beams: (a) street lamp, (b) bridge with tapered girders and piers, (c) wheel strut of a small airplane, and (d) wrench handle



Fully Stressed Beams

To minimize the amount of material and thereby have the lightest possible beam, vary the dimensions of the cross sections to have the maximum allowable bending stress at every section. A beam in this condition is called a **fully stressed beam**, or a *beam of constant strength*.

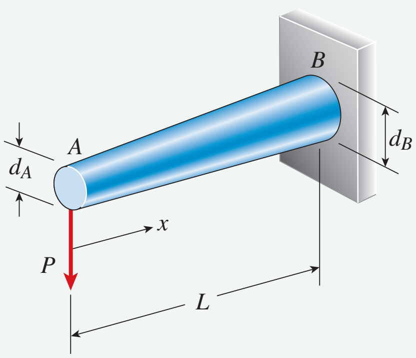
Of course, these ideal conditions are seldom attained because of practical problems in constructing the beam and the possibility of the loads being different from those assumed in design. Nevertheless, knowing the properties of a fully stressed beam can be an important aid when designing structures for minimum weight. Familiar examples of structures designed to maintain nearly constant maximum stress are leaf springs in automobiles, bridge girders that are tapered, and some of the structures shown in Fig. 5-25.

The determination of the shape of a fully stressed beam is illustrated in Example 5-10.

Example 5-9

FIGURE 5-26

Example 5-9: Tapered cantilever beam of circular cross section



A tapered cantilever beam AB with a solid circular cross section supports a load P at the free end (Fig. 5-26). The diameter d_B at the large end is twice the diameter d_A at the small end:

$$\frac{d_B}{d_A} = 2$$

Determine the bending stress σ_B at the fixed support and the maximum bending stress σ_{max}.

Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. ***Conceptualize, Categorize*:** If the angle of taper of the beam is small, the bending stresses obtained from the flexure formula differ only slightly from the exact values. As a guideline concerning accuracy, note that if the angle between line AB (Fig. 5-26) and the longitudinal axis of the beam is about 20°, the error in calculating the normal stresses from the flexure formula is about 10%. Of course, as the angle of taper decreases, the error becomes smaller.

3. ***Analyze*:**

Section modulus: The section modulus at any cross section of the beam can be expressed as a function of the distance x measured along the axis of the beam. Since the section modulus depends upon the diameter, first express the diameter in terms of x, as

$$d_x = d_A + (d_B - d_A)\frac{x}{L} \tag{5-31}$$

in which d_x is the diameter at distance x from the free end. Therefore, the section modulus at distance x from the end [Eq. (5-20b)] is

$$S_x = \frac{\pi d_x^3}{32} = \frac{\pi}{32}\left[d_A + (d_B - d_A)\frac{x}{L}\right]^3 \tag{5-32}$$

Bending stresses: Since the bending moment equals Px, the maximum normal stress at any cross section is given by

$$\sigma_1 = \frac{M_x}{S_x} = \frac{32Px}{\pi[d_A + (d_B - d_A)(x/L)]^3} \tag{5-33}$$

The stress σ_1 is tensile at the top of the beam and compressive at the bottom.

Note that Eqs. (5-31), (5-32), and (5-33) are valid for any values of d_A and d_B, provided the angle of taper is small. In the following, consider only the case where $d_B = 2d_A$.

Maximum stress at the fixed support: The maximum stress at the section of largest bending moment (end B of the beam) is obtained using Eq. (5-33) and substituting $x = L$ and $d_B = 2d_A$; the result is

$$\sigma_B = \frac{4PL}{\pi d_A^3} \tag{a}$$

Maximum stress in the beam: The maximum stress at a cross section at distance x from the end [Eq. (5-33)] assuming that $d_B = 2d_A$ is

$$\sigma_1 = \frac{32Px}{\pi d_A^3(1 + x/L)^3} \tag{b}$$

To determine the location of the cross section having the largest bending stress in the beam, find the value of x that makes σ_1 a maximum. Take the derivative $d\sigma_1/dx$ and equate it to zero, then solve for the value of x that makes σ_1 a maximum; the result is

$$x = \frac{L}{2} \tag{c}$$

The corresponding maximum stress, obtained by substituting $x = L/2$ into Eq. (b), is

$$\sigma_{max} = \frac{128PL}{27\pi d_A^3} = \frac{4.741PL}{\pi d_A^3} \tag{d}$$

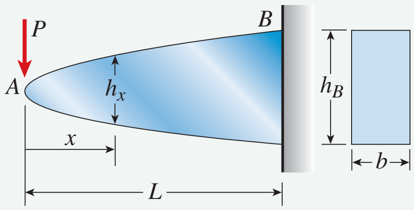
4. ***Finalize***: In this particular example, the maximum stress occurs at the midpoint of the beam and is 19% greater than the stress σ_B at the built-in end.

Note: If the taper of the beam is reduced, the cross section of maximum normal stress moves from the midpoint toward the fixed support. For small angles of taper, the maximum stress occurs at end B.

Example 5-10

FIGURE 5-27

Example 5-10: Fully stressed beam having constant maximum normal stress (theoretical shape with shear stresses disregarded)



A cantilever beam AB of length L is being designed to support a concentrated load P at the free end (Fig. 5-27). The cross sections of the beam are rectangular with a constant width b and varying height h. To assist in designing this beam, the designers want to know how the height of an idealized beam should vary in order that the maximum normal stress at every cross section will be equal to the allowable stress σ_{allow}.

Considering only the bending stresses obtained from the flexure formula, determine the height of the fully stressed beam.

Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. ***Conceptualize, Categorize***: The bending moment and section modulus at distance x from the free end of the beam are

$$M = Px \qquad S = \frac{bh_x^2}{6}$$

where h_x is the height of the beam at distance x. Substitute in the flexure formula to obtain

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{bh_x^2/6} = \frac{6Px}{bh_x^2} \tag{a}$$

3. ***Analyze***: Solve for the height of the beam to find

$$h_x = \sqrt{\frac{6Px}{b\sigma_{\text{allow}}}} \tag{b}$$

At the fixed end of the beam ($x = L$), the height h_B is

$$h_B = \sqrt{\frac{6PL}{b\sigma_{\text{allow}}}} \tag{c}$$

therefore, the height h_x is expressed as

$$h_x = h_B \sqrt{\frac{x}{L}} \qquad \text{(d)}$$

4. ***Finalize***: This last equation shows that the height of the fully stressed beam varies with the square root of x. Consequently, the idealized beam has the parabolic shape shown in Fig. 5-27.

Note: At the loaded end of the beam ($x = 0$), the theoretical height is zero because there is no bending moment at that point. A beam of this shape is not practical because it is incapable of supporting the shear forces near the end of the beam. Nevertheless, the idealized shape can provide a useful starting point for a realistic design in which shear stresses and other effects are considered.

5.8 Shear Stresses in Beams of Rectangular Cross Section

When a beam is in *pure bending*, the only stress resultants are the bending moments and the only stresses are the normal stresses acting on the cross sections. However, most beams are subjected to loads that produce both bending moments and shear forces (*nonuniform bending*). In these cases, both normal and shear stresses are developed in the beam. The normal stresses are calculated from the flexure formula (see Section 5.5), provided the beam is constructed of a linearly elastic material. The shear stresses are discussed in this and the following two sections.

Vertical and Horizontal Shear Stresses

Consider a beam of rectangular cross section (width b and height h) subjected to a positive shear force V (Fig. 5-28a). It is reasonable to assume that the shear stresses τ acting on the cross section are parallel to the shear force, that is, parallel to the vertical sides of the cross section. It is also reasonable to assume that the shear stresses are uniformly distributed across the width of the beam, although they may vary over the height. Using these two assumptions, you can determine the intensity of the shear stress at any point on the cross section.

For purposes of analysis, isolate a small element mn of the beam (Fig. 5-28a) by cutting between two adjacent cross sections and between two horizontal planes. Assume the shear stresses τ acting on the front face of this element are vertical and uniformly distributed from one side of the beam to the other. Also, from the discussion of shear stresses in Section 1.8, shear stresses acting on one side of an element are accompanied by shear stresses of equal magnitude acting on perpendicular faces of the element (Figs. 5-28b and c). Thus, there are horizontal shear stresses acting between horizontal layers of the beam as well as vertical shear stresses acting on the cross sections. At any point in the beam, these complementary shear stresses are equal in magnitude.

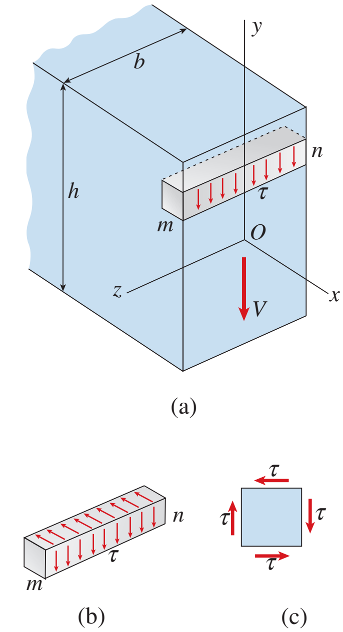
The equality of the horizontal and vertical shear stresses acting on an element leads to an important conclusion regarding the shear stresses at the top and bottom of the beam. If you imagine that the element mn (Fig. 5-28a) is located at either the top or the bottom, it follows that the horizontal shear stresses must vanish, because there are no stresses on the outer surfaces of the beam. It follows that the vertical shear stresses must also vanish at those locations; in other words, $\tau = 0$ where $y = \pm h/2$.

The existence of horizontal shear stresses in a beam can be demonstrated by a simple experiment. Place two identical rectangular beams on simple supports and load them by a force P, as shown in Fig. 5-29a. If friction between the beams is small, the beams bend independently (Fig. 5-29b). Each beam is in compression above its own neutral axis and in tension below its neutral axis; therefore, the bottom surface of the upper beam slides with respect to the top surface of the lower beam.

Now suppose that the two beams are glued along the contact surface, so they become a single solid beam. When this beam is loaded, horizontal shear stresses must develop along the glued surface in order to prevent the sliding shown in Fig. 5-29b. Because of the presence of these shear stresses, the single solid beam is much stiffer and stronger than the two separate beams.

FIGURE 5-28

Shear stresses in a beam of rectangular cross section



Derivation of Shear Formula

Now derive a formula for the shear stresses τ in a rectangular beam. However, instead of evaluating the vertical shear stresses acting on a cross section, it is easier to evaluate the horizontal shear stresses acting between layers of the beam. Of course, the vertical shear stresses have the same magnitudes as the horizontal shear stresses.

Now consider a beam in nonuniform bending (Fig. 5-30a). Take two adjacent cross sections mn and m_1n_1 at a distance dx apart, and consider the **element** mm_1n_1n. The bending moment and shear force acting on the left-hand face of this element are denoted M and V, respectively. Since both the bending moment and shear force may change when moving along the axis of the beam, the corresponding quantities on the right-hand face (Fig. 5-30a) are denoted $M + dM$ and $V + dV$.

Because of the presence of the bending moments and shear forces, the element shown in Fig. 5-30a is subjected to normal and shear stresses on both cross-sectional faces. However, only the normal stresses are needed in the following derivation, so only the normal stresses are shown in Fig. 5-30b. On cross sections mn and m_1n_1, the normal stresses are, respectively,

$$\sigma_1 = -\frac{My}{I} \quad \text{and} \quad \sigma_2 = -\frac{(M + dM)y}{I} \qquad \textbf{(5-34a,b)}$$

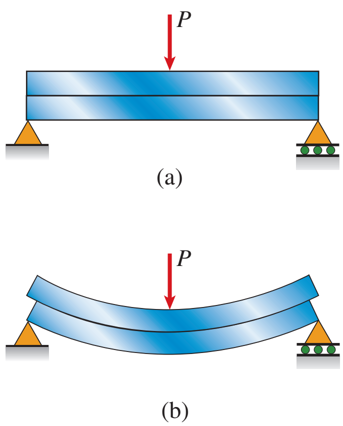
as given by the flexure formula [Eq. (5-14)]. In these expressions, y is the distance from the neutral axis and I is the moment of inertia of the cross-sectional area about the neutral axis.

Next, isolate a **subelement** mm_1p_1p by passing a horizontal plane pp_1 through element mm_1n_1n (Fig. 5-30b). The plane pp_1 is at distance y_1 from the neutral surface of the beam. The subelement is shown separately in Fig. 5-30c. Note that its top face is part of the upper surface of the beam and thus is free from stress. Its bottom face (which is parallel to the neutral surface and distance y_1 from it) is acted upon by the horizontal shear stresses τ existing at this level in the beam. Its cross-sectional faces mp and m_1p_1 are acted upon by the bending stresses σ_1 and σ_2, respectively, which are produced by the bending moments. Vertical shear stresses also act on the cross-sectional faces; however, these stresses do not affect the equilibrium of the subelement in the horizontal direction (the x direction), so they are not shown in Fig. 5-30c.

If the bending moments at cross sections mn and m_1n_1 (Fig. 5-30b) are equal (that is, if the beam is in pure bending), the normal stresses σ_1 and σ_2 acting

FIGURE 5-29

Bending of two separate beams



over the sides mp and m_1p_1 of the subelement (Fig. 5-30c) also are equal. Under these conditions, the subelement is in equilibrium under the action of the normal stresses alone; therefore, the shear stresses τ acting on the bottom face pp_1 vanish. This conclusion is obvious inasmuch as a beam in pure bending has no shear force and hence no shear stresses.

If the bending moments vary along the x axis (nonuniform bending), the shear stress τ acting on the bottom face of the subelement (Fig. 5-30c) can be determined by considering the equilibrium of the subelement in the x direction.

Begin by identifying an element of area dA in the *cross section* at a distance y from the neutral axis (Fig. 5-30d). The force acting on this element is σdA, in which σ is the normal stress obtained from the flexure formula. If the element of area is located on the left-hand face mp of the subelement (where the bending moment is M), the normal stress is given by Eq. (5-34a); therefore, the element of force is

$$\sigma_1 dA = \frac{My}{I} dA$$

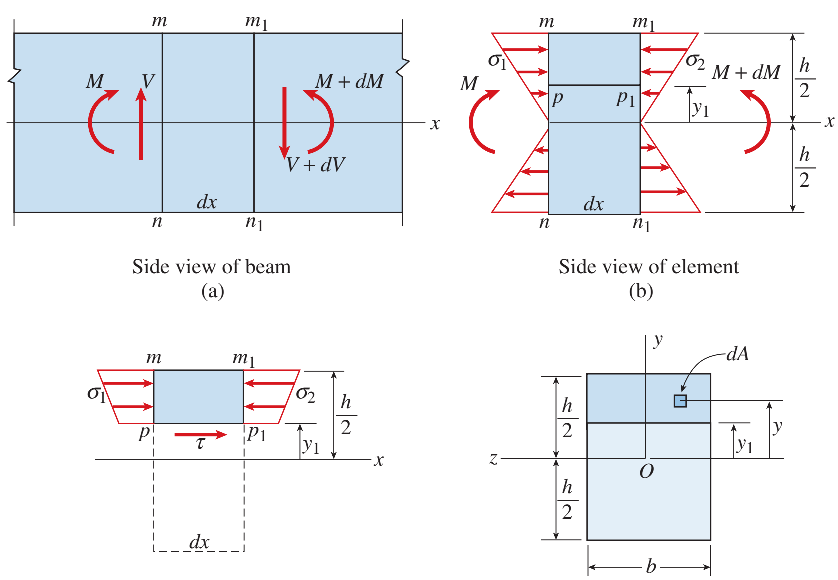
Note that only absolute values are used in this equation because the directions of the stresses are obvious from Fig. 5-30. Summing these elements of force over the area of face mp of the subelement (Fig. 5-30c) gives the total horizontal force F_1 acting on that face:

$$F_1 = \int \sigma_1 dA = \int \frac{My}{I} dA \qquad \textbf{(5-35a)}$$

Note that this integration is performed over the area of the shaded part of the cross section shown in Fig. 5-30d, that is, over the area of the cross section from $y = y_1$ to $y = h/2$.

The force F_1 is shown in Fig. 5-31 on a partial free-body diagram of the subelement. (Vertical forces have been omitted.)

FIGURE 5-30
Shear stresses in a beam of rectangular cross section



Side view of beam (a)

Side view of element (b)

Side view of subelement (c)

Cross section of beam at subelement (d)

In a similar manner, the total force F_2 acting on the right-hand face m_1p_1 of the subelement (Fig. 5-31 and Fig. 5-30c) is

$$F_2 = \int \sigma_2 dA = \int \frac{(M + dM)y}{I} dA \tag{5-35b}$$

Knowing the forces F_1 and F_2, now determine the horizontal force F_3 acting on the bottom face of the subelement.

Since the subelement is in equilibrium, sum forces in the x direction and obtain

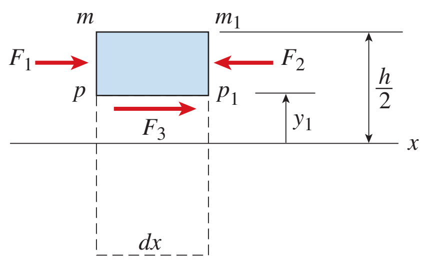
$$F_3 = F_2 - F_1 \tag{5-35c}$$

or

$$F_3 = \int \frac{(M + dM)y}{I} dA - \int \frac{My}{I} dA = \int \frac{(dM)y}{I} dA$$

FIGURE 5-31

Partial free-body diagram of subelement showing all horizontal forces (compare with Fig. 5-30c)



The quantities dM and I in the last term can be moved outside the integral sign because they are constants at any given cross section and are not involved in the integration. Thus, the expression for the force F_3 becomes

$$F_3 = \frac{dM}{I} \int y dA \tag{5-36}$$

If the shear stresses τ are uniformly distributed across the width b of the beam, the force F_3 is also equal to

$$F_3 = \tau b dx \tag{5-37}$$

in which bdx is the area of the bottom face of the subelement.

Combine Eqs. (5-36) and (5-37) and solve for the shear stress τ to get

$$\tau = \frac{dM}{dx}\left(\frac{1}{Ib}\right)\int y dA \tag{5-38}$$

The quantity dM/dx is equal to the shear force V (see Eq. 4-4), so the preceding expression becomes

$$\tau = \frac{V}{Ib}\int y dA \tag{5-39}$$

The integral in this equation is evaluated over the shaded part of the cross section (Fig. 5-30d), as already explained. Thus, the integral is the first moment of the shaded area with respect to the neutral axis (the z axis). In other words, *the integral is the first moment of the cross-sectional area above the level at which the shear stress τ is being evaluated*. This first moment is usually denoted by the symbol Q:

$$Q = \int y dA \tag{5-40}$$

With this notation, the equation for the shear stress becomes

$$\tau = \frac{VQ}{Ib} \tag{5-41}$$

This equation, known as the **shear formula**, can be used to determine the shear stress τ at any point in the cross section of a rectangular beam. Note that for a specific cross section, the shear force V, moment of inertia I, and width b are constants. However, the first moment Q (and hence the shear stress τ) varies with the distance y_1 from the neutral axis.

Calculation of the First Moment Q

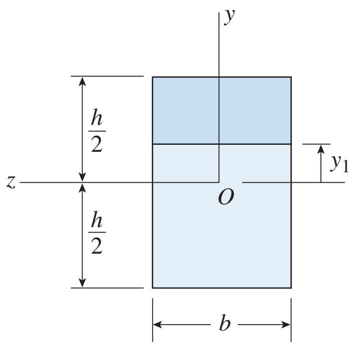
If the level at which the shear stress is to be determined is above the neutral axis, as shown in Fig. 5-30d, it is natural to obtain Q by calculating the first moment of the cross-sectional area *above* that level (the shaded area in the figure). However, as an alternative, you could calculate the first moment of the remaining cross-sectional area, that is, the area *below* the shaded area. Its first moment is equal to the negative of Q.

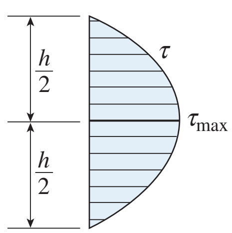
The explanation lies in the fact that the first moment of the entire cross-sectional area with respect to the neutral axis is equal to zero (because the neutral axis passes through the centroid). Therefore, the value of Q for the area below the level y_1 is the negative of Q for the area above that level. Use the area above the level y_1 when the point where the shear stress is computed is in the upper part of the beam, and use the area below the level y_1 when the point is in the lower part of the beam.

Furthermore, don't bother with sign conventions for V and Q. Instead, treat all terms in the shear formula as positive quantities and determine the direction of the shear stresses by inspection, since the stresses act in the same direction as the shear force V itself. This procedure for determining shear stresses is illustrated in Example 5-11.

FIGURE 5-32

Distribution of shear stresses in a beam of rectangular cross section: (a) cross section of beam and (b) diagram showing the parabolic distribution of shear stresses over the height of the beam





Distribution of Shear Stresses in a Rectangular Beam

Now find the distribution of the shear stresses in a beam of rectangular cross section (Fig. 5-32). Obtain the first moment Q of the shaded part of the cross-sectional area by multiplying the area by the distance from its own centroid to the neutral axis:

$$Q = b\left(\frac{h}{2} - y_1\right)\left(y_1 + \frac{h/2 - y_1}{2}\right) = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right) \tag{5-42a}$$

This same result can be obtained by integration using Eq. (5-40):

$$Q = \int y\,dA = \int_0^{h/2} ybdy = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right) \tag{5-42b}$$

Substitute the expression for Q into the shear formula [Eq. (5-41)] to get

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right) \tag{5-43}$$

This equation shows that the shear stresses in a rectangular beam vary quadratically with the distance y_1 from the neutral axis. Thus, when plotted along the height of the beam, τ varies as shown in Fig. 5-32b. Note that the shear stress is zero when $y_1 = \pm h/2$.

The maximum value of the shear stress occurs at the neutral axis ($y_1 = 0$) where the first moment Q has its maximum value. Substitute $y_1 = 0$ into Eq. (5-43) to get

$$\tau_{\max} = \frac{Vh^2}{8I} = \frac{3V}{2A} \tag{5-44}$$

in which $A = bh$ is the cross-sectional area. Thus, the maximum shear stress in a beam of rectangular cross section is 50% larger than the average shear stress V/A.

Note again that the preceding equations for the shear stresses can be used to calculate either the vertical shear stresses acting on the cross sections or the horizontal shear stresses acting between horizontal layers of the beam.[5]

Limitations

The formulas for shear stresses in this section are subject to the same restrictions as the flexure formula from which they are derived. Thus, they are valid only for beams of linearly elastic materials with small deflections.

In the case of rectangular beams, the accuracy of the shear formula depends upon the height-to-width ratio of the cross section. The formula may be considered as exact for very narrow beams (height h much larger than the width b). However, it becomes less accurate as b increases relative to h. For instance, when the beam is square ($b = h$), the true maximum shear stress is about 13% larger than the value given by Eq. (5-44). (For a more complete discussion of the limitations of the shear formula, see Ref. 5-9.)

A common error is to apply the shear formula [(Eq. (5-41)] to cross-sectional shapes for which it is not applicable. For instance, it is not applicable to sections of triangular or semicircular shapes. To avoid misusing the formula, keep in mind the following assumptions that underlie the derivation: (1) The edges of the cross section must be parallel to the y axis (so that the shear stresses act parallel to the y axis), and (2) the shear stresses must be uniform across the width of the cross section. These assumptions are fulfilled only in certain cases, such as those discussed in this and the next two sections.

Finally, the shear formula applies only to prismatic beams. If a beam is nonprismatic (for instance, if the beam is tapered), the shear stresses are quite different from those predicted by the formulas given here (see Refs. 5-9 and 5-10).

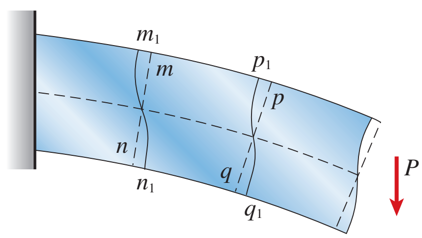
Effects of Shear Strains

Because the shear stress τ varies parabolically over the height of a rectangular beam, it follows that the shear strain $\gamma = \tau/G$ also varies parabolically. As a result of these shear strains, cross sections of the beam that were originally plane surfaces become warped. This warping is shown in Fig. 5-33, where cross sections mn and pq, originally plane, have become curved surfaces m_1n_1 and p_1q_1, with the maximum shear strain occurring at the neutral surface. At points m_1, p_1, n_1, and q_1, the shear strain is zero, and therefore the curves m_1n_1 and p_1q_1 are perpendicular to the upper and lower surfaces of the beam.

If the shear force V is constant along the axis of the beam, warping is the same at every cross section. Therefore, stretching and shortening of longitudinal elements due to the bending moments is unaffected by the shear strains, and the distribution of the normal stresses is the same as in pure bending. Moreover, detailed investigations using advanced methods of analysis show that the warping of cross sections due to shear strains does not substantially affect the longitudinal strains even when the shear force varies continuously along the length. Thus, under most conditions, it is justifiable to use the flexure formula [Eq. (5-14)] for nonuniform bending, even though the formula was derived for pure bending.

FIGURE 5-33

Warping of the cross sections of a beam due to shear strains

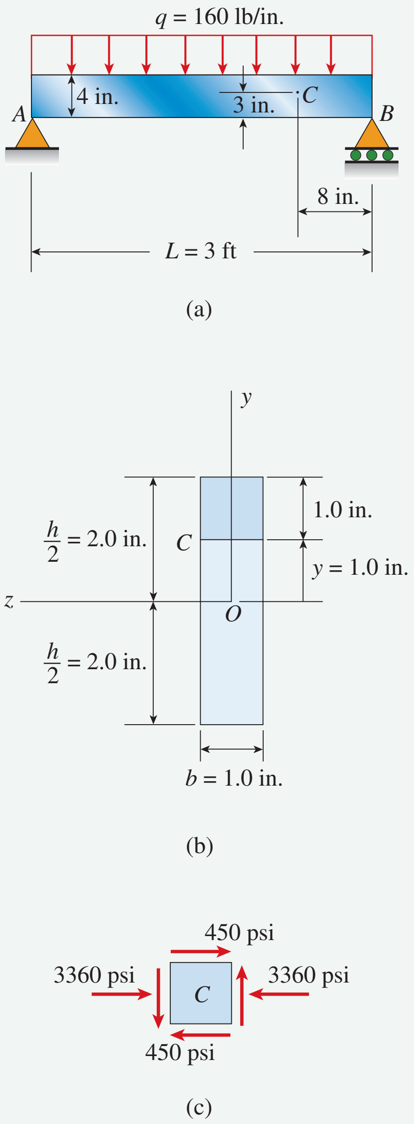


[5]The shear-stress analysis presented in this section was developed by the Russian engineer D. J. Jourawski; see Refs. 5-7 and 5-8.

Example 5-11

FIGURE 5-34

Example 5-11: (a) Simple beam with uniform load, (b) cross section of beam, and (c) stress element showing the normal and shear stresses at point C



A metal beam with a span $L = 3$ ft is simply supported at points A and B (Fig. 5-34a). The uniform load on the beam (including its own weight) is $q = 160$ lb/in. The cross section of the beam is rectangular (Fig. 5-34b) with width $b = 1$ in. and height $h = 4$ in. The beam is adequately supported against sideways buckling.

Determine the normal stress σ_C and shear stress τ_C at point C, which is located 1 in. below the top of the beam and 8 in. from the right-hand support. Show these stresses on a sketch of a stress element at point C.

Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. ***Conceptualize, Categorize*:**

Shear force and bending moment: The shear force V_C and bending moment M_C at the cross section through point C are found as described in Chapter 4. The results are

$$M_C = 17{,}920 \text{ lb-in.} \qquad V_C = -1600 \text{ lb}$$

The signs of these quantities are based upon the standard sign conventions for bending moments and shear forces (see Fig. 4-19).

Moment of inertia: The moment of inertia of the cross-sectional area about the neutral axis (the z axis in Fig. 5-34b) is

$$I = \frac{bh^3}{12} = \frac{1}{12}(1.0 \text{ in.})(4.0 \text{ in.})^3 = 5.333 \text{ in}^4$$

3. ***Analyze*:**

Normal stress at point C: The normal stress at point C is found from the flexure formula [Eq. (5-14)] with the distance y from the neutral axis equal to 1.0 in.; thus,

$$\sigma_C = -\frac{My}{I} = -\frac{(17{,}920 \text{ lb-in.})(1.0 \text{ in.})}{5.333 \text{ in}^4} = -3360 \text{ psi}$$

The minus sign indicates that the stress is compressive, as expected.

Shear stress at point C: To obtain the shear stress at point C, evaluate the first moment Q_C of the cross-sectional area above point C (Fig. 5-34b). This first moment is equal to the product of the area and its centroidal distance (denoted y_C) from the z axis; thus,

$$A_C = (1.0 \text{ in.})(1.0 \text{ in.}) = 1.0 \text{ in}^2 \quad y_C = 1.5 \text{ in.} \quad Q_C = A_C y_C = 1.5 \text{ in}^3$$

Now substitute numerical values into the shear formula [Eq. (5-41)] and obtain the magnitude of the shear stress:

$$\tau_C = \frac{V_C Q_C}{Ib} = \frac{(1600 \text{ lb})(1.5 \text{ in}^3)}{(5.333 \text{ in}^4)(1.0 \text{ in.})} = 450 \text{ psi}$$

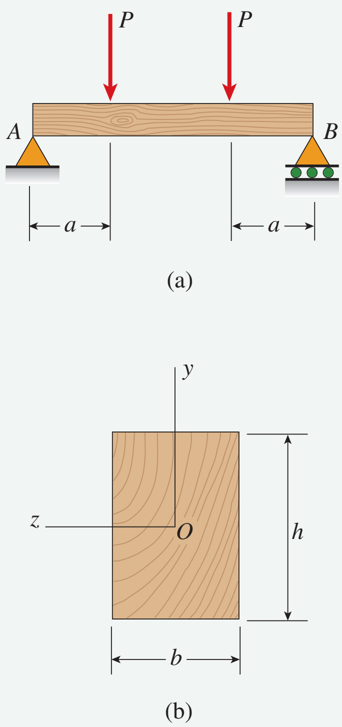
4. ***Finalize***: The direction of this stress can be established by inspection because it acts in the same direction as the shear force. In this example, the shear force acts upward on the part of the beam to the left of point *C* and downward on the part of the beam to the right of point *C*. The best way to show the directions of both the normal and shear stresses is to draw a stress element.

Stress element at point *C*: The stress element, shown in Fig. 5-34c, is cut from the side of the beam at point *C* (Fig. 5-34a). Compressive stresses $\sigma_C = 3360$ psi act on the cross-sectional faces of the element and shear stresses $\tau_C = 450$ psi act on the top and bottom faces as well as the cross-sectional faces.

Example 5-12

FIGURE 5-35

Example 5-12: Wood beam with concentrated loads



A wood beam *AB* supporting two concentrated loads *P* (Fig. 5-35a) has a rectangular cross section of width $b = 100$ mm and height $h = 150$ mm (Fig. 5-35b). The distance from each end of the beam to the nearest load is $a = 0.5$ m.

Determine the maximum permissible value P_{max} of the loads if the allowable stress in bending is $\sigma_{allow} = 11$ MPa (for both tension and compression) and the allowable stress in horizontal shear is $\tau_{allow} = 1.2$ MPa. (Disregard the weight of the beam itself.)

Note: Wood beams are much weaker in *horizontal shear* (shear parallel to the longitudinal fibers in the wood) than in *cross-grain shear* (shear on the cross sections). Consequently, the allowable stress in horizontal shear is usually considered in design.

Solution:

Use a four-step problem-solving approach.

1. ***Conceptualize***: The maximum shear force occurs at the supports, and the maximum bending moment occurs throughout the region between the loads. Their values are

$$V_{max} = P \quad M_{max} = Pa$$

Also, the section modulus *S* and cross-sectional area *A* are

$$S = \frac{bh^2}{6} \quad A = bh$$

2. ***Categorize***: The maximum normal and shear stresses in the beam are obtained from the flexure and shear formulas [Eqs. (5-17) and (5-44)]:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{6Pa}{bh^2} \qquad \tau_{max} = \frac{3V_{max}}{2A} = \frac{3P}{2bh}$$

Therefore, the maximum permissible values of the load P in bending and shear, respectively, are

$$P_{bending} = \frac{\sigma_{allow}bh^2}{6a} \qquad P_{shear} = \frac{2\tau_{allow}bh}{3}$$

3. ***Analyze***: Substitute numerical values into these formulas to get

$$P_{bending} = \frac{(11\text{ MPa})(100\text{ mm})(150\text{ mm})^2}{6(0.5\text{ m})} = 8.25\text{ kN}$$

$$P_{shear} = \frac{2(1.2\text{ MPa})(100\text{ mm})(150\text{ mm})}{3} = 12.0\text{ kN}$$

Thus, the bending stress governs the design, and the maximum permissible load is

$$P_{max} = 8.25\text{ kN}$$

4. ***Finalize***: A more complete analysis of this beam would require that the weight of the beam be taken into account, thus reducing the permissible load.

Notes:

i. In this example, the maximum normal stresses and maximum shear stresses do not occur at the same locations in the beam—the normal stress is maximum in the middle region of the beam at the top and bottom of the cross section, and the shear stress is maximum near the supports at the neutral axis of the cross section.

ii. For most beams, the bending stresses (not the shear stresses) control the allowable load, as in this example.

iii. Although wood is not a homogeneous material and often departs from linearly elastic behavior, approximate results still can be obtained from the flexure and shear formulas. These approximate results are usually adequate for designing wood beams.

5.9 Shear Stresses in Beams of Circular Cross Section

When a beam has a **circular cross section** (Fig. 5-36), you can no longer assume that the shear stresses act parallel to the y axis. For instance, it is easy to prove that at point m (on the boundary of the cross section) the shear stress τ must act *tangent* to the boundary. This observation follows from the fact that the outer surface of the beam is free of stress, and the shear stress acting on the cross section can have no component in the radial direction.

Although there is no simple way to find the shear stresses acting throughout the entire cross section, the shear stresses at the neutral axis (where the stresses are the largest) are found by making some reasonable assumptions about the stress distribution. Assume that the stresses act parallel to the y axis and have a constant intensity across the width of the beam (from point p to point q in Fig. 5-36). Since these assumptions are the same as those used in deriving the shear formula $\tau = VQ/Ib$ [Eq. (5-41)], use the shear formula to calculate the stresses at the neutral axis.

For use in the shear formula, the following properties pertaining to a circular cross section having radius r are needed:

$$I = \frac{\pi r^4}{4} \qquad Q = A\bar{y} = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) = \frac{2r^3}{3} \qquad b = 2r \qquad \textbf{(5-45a,b)}$$

The expression for the moment of inertia I is taken from Case 9 of Appendix E, and the expression for the first moment Q is based upon the formulas for a semicircle (Case 10, Appendix E). Substitute these expressions into the shear formula to obtain

$$\tau_{max} = \frac{VQ}{Ib} = \frac{V(2r^3/3)}{(\pi r^4/4)(2r)} = \frac{4V}{3\pi r^2} = \frac{4V}{3A} \qquad \textbf{(5-46)}$$

in which $A = \pi r^2$ is the area of the cross section. This equation shows that the maximum shear stress in a circular beam is equal to 4/3 times the average vertical shear stress V/A.

For a beam with a **hollow circular cross section** (Fig. 5-37), again assume with reasonable accuracy that the shear stresses at the neutral axis are parallel to the y axis and uniformly distributed across the section. Consequently, the shear formula is used to find the maximum stresses. The required properties for a hollow circular section are

$$I = \frac{\pi}{4}(r_2^4 - r_1^4) \qquad Q = \frac{2}{3}(r_2^3 - r_1^3) \qquad b = 2(r_2 - r_1) \qquad \textbf{(5-47a,b,c)}$$

in which r_1 and r_2 are the inner and outer radii of the cross section. Therefore, the maximum stress is

$$\tau_{max} = \frac{VQ}{Ib} = \frac{4V}{3A}\left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}\right) \qquad \textbf{(5-48)}$$

in which

$$A = \pi(r_2^2 - r_1^2)$$

is the area of the cross section. Note that if $r_1 = 0$, Eq. (5-48) reduces to Eq. (5-46) for a solid circular beam.

Although the preceding theory for shear stresses in beams of circular cross section is approximate, it gives results differing by only a few percent from those obtained using the exact theory of elasticity (Ref. 5-9). Consequently, Eqs. (5-46) and (5-48) can be used to determine the maximum shear stresses in circular beams under ordinary circumstances.

FIGURE 5-36

Shear stresses acting on the cross section of a circular beam

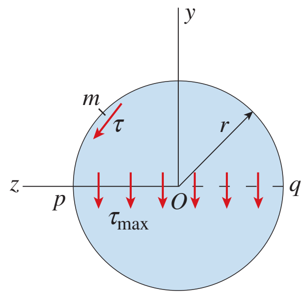
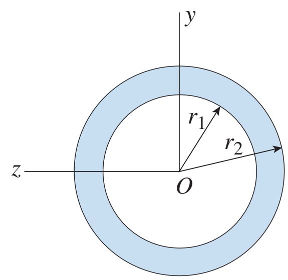


FIGURE 5-37

Hollow circular cross section



Example 5-13

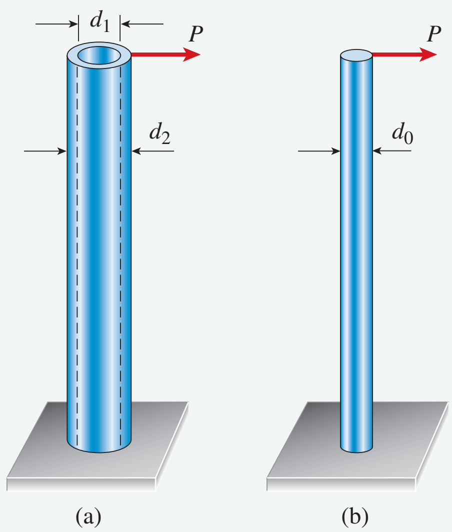
A vertical pole consisting of a circular tube of outer diameter $d_2 = 4.0$ in. and inner diameter $d_1 = 3.2$ in. is loaded by a horizontal force $P = 1500$ lb (Fig. 5-38a).

(a) Determine the maximum shear stress in the pole.

(b) For the same load P and the same maximum shear stress, what is the diameter d_0 of a solid circular pole (Fig. 5-38b)?

FIGURE 5-38

Example 5-13: Shear stresses in beams of circular cross section



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

Part (a): Maximun shear stress.

1, 2. ***Conceptualize, Categorize***: For the pole having a hollow circular cross section (Fig. 5-38a), use Eq. (5-48) with the shear force V replaced by the load P and the cross-sectional area A replaced by the expression $\pi(r_2^2 - r_1^2)$; thus,

$$\tau_{max} = \frac{4P}{3\pi}\left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^4 - r_1^4}\right) \qquad \textbf{(a)}$$

3, 4. ***Analyze, Finalize***: Next, substitute numerical values, namely,

$$P = 1500 \text{ lb} \qquad r_2 = d_2/2 = 2.0 \text{ in.} \qquad r_1 = d_1/2 = 1.6 \text{ in.}$$

to obtain

$$\tau_{max} = 658 \text{ psi}$$

which is the maximum shear stress in the pole.

Part (b): Diameter of solid circular pole.

1, 2. ***Conceptualize, Categorize*****:** For the pole having a solid circular cross section (Fig. 5-36b), use Eq. (5-46) with V replaced by P and r replaced by $d_0/2$:

$$\tau_{max} = \frac{4P}{3\pi(d_0/2)^2} \tag{b}$$

3. ***Analyze*****:** Solve for d_0 to obtain

$$d_0^2 = \frac{16P}{3\pi\tau_{max}} = \frac{16(1500 \text{ lb})}{3\pi(658 \text{ psi})} = 3.87 \text{ in}^2$$

that produces

$$d_0 = 1.97 \text{ in.}$$

4. ***Finalize*****:** In this particular example, the solid circular pole has a diameter approximately one-half that of the tubular pole.

Note: Shear stresses rarely govern the design of either circular or rectangular beams made of metals such as steel and aluminum. In these kinds of materials, the allowable shear stress is usually in the range 25 to 50% of the allowable tensile stress. In the case of the tubular pole in this example, the maximum shear stress is only 658 psi. In contrast, the maximum bending stress obtained from the flexure formula is 9700 psi for a relatively short pole of length 24 in. Thus, as the load increases, the allowable tensile stress will be reached long before the allowable shear stress is reached.

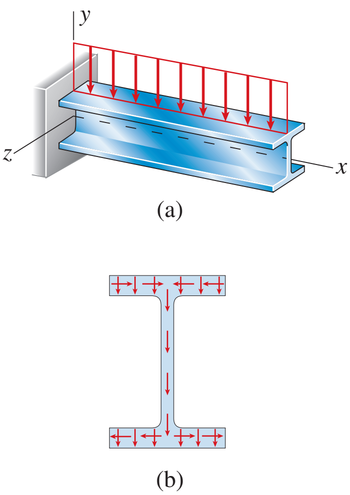
The situation is quite different for materials that are weak in shear, such as wood. For a typical wood beam, the allowable stress in horizontal shear is in the range of 4 to 10% of the allowable bending stress. Consequently, even though the maximum shear stress is relatively low in value, it sometimes governs the design.

5.10 Shear Stresses in the Webs of Beams with Flanges

When a beam of wide-flange shape (Fig. 5-39a) is subjected to shear forces as well as bending moments (nonuniform bending), both normal and shear stresses are developed on the cross sections. The distribution of the shear stresses in a wide-flange beam is more complicated than in a rectangular beam. For instance, the shear stresses in the flanges of the beam act in both vertical and horizontal directions (the y and z directions), as shown by the small arrows in Fig. 5-39b. The horizontal shear stresses are much larger than the vertical shear stresses in the flanges and are discussed later in Section 6.8.

FIGURE 5-39

(a) Beam of wide-flange shape and (b) directions of the shear stresses acting on a cross section



The shear stresses in the web of a wide-flange beam act only in the vertical direction and are larger than the stresses in the flanges. These stresses can be found by the same techniques used for finding shear stresses in rectangular beams.

Shear Stresses in the Web

Begin the analysis by determining the shear stresses at line *ef* in the web of a wide-flange beam (Fig. 5-40a). Make the same assumptions as those made for a rectangular beam; that is, assume that the shear stresses act parallel to the y axis and are uniformly distributed across the thickness of the web. Then the shear formula $\tau = VQ/Ib$ will still apply. However, the width b is now the thickness t of the web, and the area used in calculating the first moment Q is the area between line *ef* and the top edge of the cross section (indicated by the shaded area of Fig. 5-40a).

When finding the first moment Q of the shaded area, disregard the effects of the small fillets at the juncture of the web and flange (points b and c in Fig. 5-40a). The error in ignoring the areas of these fillets is very small. Then divide the shaded area into two rectangles. The first rectangle is the upper flange itself, which has the area

$$A_1 = b\left(\frac{h}{2} - \frac{h_1}{2}\right) \tag{5-49a}$$

in which b is the width of the flange, h is the overall height of the beam, and h_1 is the distance between the insides of the flanges. The second rectangle is the part of the web between *ef* and the flange, that is, rectangle *efcb*, which has the area

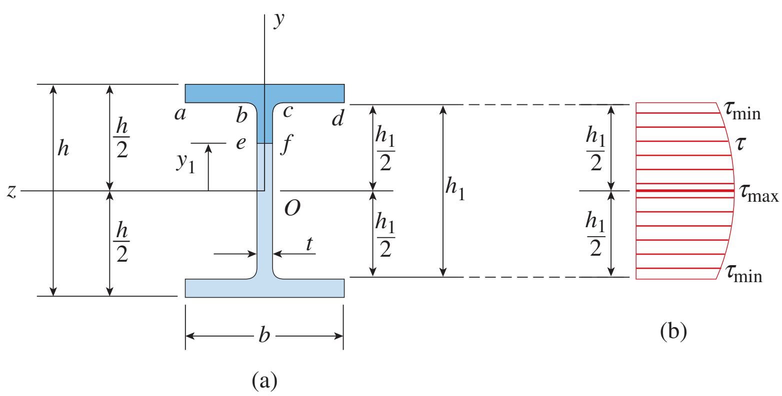
$$A_2 = t\left(\frac{h_1}{2} - y_1\right) \tag{5-49b}$$

in which t is the thickness of the web and y_1 is the distance from the neutral axis to line *ef*.

The first moments of areas A_1 and A_2, evaluated about the neutral axis, are obtained by multiplying these areas by the distances from their respective

FIGURE 5-40

Shear stresses in the web of a wide-flange beam: (a) cross section of beam and (b) distribution of vertical shear stresses in the web



centroids to the z axis. Adding these first moments gives the first moment Q of the combined area:

$$Q = A_1\left(\frac{h_1}{2} + \frac{h/2 - h_1/2}{2}\right) + A_2\left(y_1 + \frac{h_1/2 - y_1}{2}\right)$$

Substituting for A_1 and A_2 from Eqs. (5-49a and b) and then simplifying gives

$$Q = \frac{b}{8}(h^2 - h_1^2) + \frac{t}{8}(h_1^2 - 4y_1^2) \tag{5-50}$$

Therefore, the shear stress τ in the web of the beam at distance y_1 from the neutral axis is

$$\tau = \frac{VQ}{It} = \frac{V}{8It}\left[b(h^2 - h_1^2) + t(h_1^2 - 4y_1^2)\right] \tag{5-51}$$

in which the moment of inertia of the cross section is

$$I = \frac{bh^3}{12} - \frac{(b - t)h_1^3}{12} = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) \tag{5-52}$$

Since all quantities in Eq. (5-51) are constants except y_1, note that τ varies quadratically throughout the height of the web, as shown by the graph in Fig. 5-40b. The graph is drawn only for the web and does not include the flanges. The reason is simple enough—Eq. (5-51) cannot be used to determine the vertical shear stresses in the flanges of the beam (see the discussion titled "Limitations" later in this section).

Maximum and Minimum Shear Stresses

The maximum shear stress in the web of a wide-flange beam occurs at the neutral axis where $y_1 = 0$. The minimum shear stress occurs where the web meets the flanges ($y_1 = \pm h_1/2$). These stresses, found from Eq. (5-51), are

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) \qquad \tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) \tag{5-53a,b}$$

Both τ_{max} and τ_{min} are labeled on the graph of Fig. 5-40b. For typical wide-flange beams, the maximum stress in the web is from 10 to 60% greater than the minimum stress.

Although it may not be apparent from the preceding discussion, the stress τ_{max} given by Eq. (5-53a) not only is the largest shear stress in the web but also is the largest shear stress anywhere in the cross section.

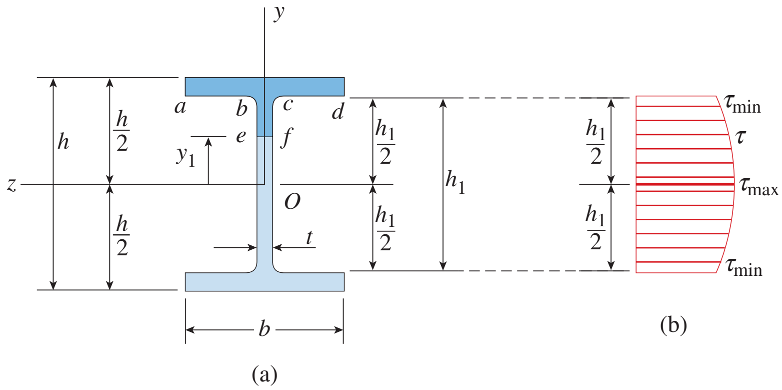
Shear Force in the Web

The vertical shear force carried by the web alone may be determined by multiplying the area of the shear-stress diagram (Fig. 5-40b) by the thickness t of the web. The shear-stress diagram consists of two parts: a rectangle of area $h_1\tau_{min}$ and a parabolic segment of area

$$\frac{2}{3}(h_1)(\tau_{max} - \tau_{min})$$

FIGURE 5-40 (Repeated)

Shear stresses in the web of a wide-flange beam: (a) cross section of beam and (b) distribution of vertical shear stresses in the web



Adding these two areas, multiplying by the thickness t of the web, and then combining terms gives the total shear force in the web:

$$V_{\text{web}} = \frac{th_1}{3}(2\tau_{\max} + \tau_{\min}) \tag{5-54}$$

For beams of typical proportions, the shear force in the web is 90 to 98% of the total shear force V acting on the cross section; the remainder is carried by shear in the flanges.

Since the web resists most of the shear force, designers often calculate an approximate value of the maximum shear stress by dividing the total shear force by the area of the web. The result is the average shear stress in the web, assuming that the web carries *all* of the shear force:

$$\tau_{\text{aver}} = \frac{V}{th_1} \tag{5-55}$$

For typical wide-flange beams, the average stress calculated in this manner is within 10% (plus or minus) of the maximum shear stress calculated from Eq. (5-53a). Thus, Eq. (5-55) provides a simple way to estimate the maximum shear stress.

Limitations

The elementary shear theory presented in this section is suitable for determining the vertical shear stresses in the web of a wide-flange beam. However, when investigating vertical shear stresses in the flanges, you can no longer assume that the shear stresses are constant across the width of the section, that is, across the width b of the flanges (Fig. 5-40a). Hence, you cannot use the shear formula to determine these stresses.

To emphasize this point, consider the junction of the web and upper flange ($y_1 = h_1/2$), where the width of the section changes abruptly from t to b. The shear stresses on the free surfaces ab and cd (Fig. 5-40a) must be zero, whereas the shear stress across the web at line bc is $\tau_{\min}$. These observations indicate that the distribution of shear stresses at the junction of the web and the flange is quite complex and cannot be investigated by elementary methods. The stress analysis is further complicated by the use of fillets at the re-entrant corners (corners b and c). The fillets are necessary to prevent the stresses from becoming dangerously large, but they also alter the stress distribution across the web.

Thus, the shear formula cannot be used to determine the vertical shear stresses in the flanges. However, the shear formula does give good results for the shear stresses acting *horizontally* in the flanges (Fig. 5-39b), as discussed later in Section 6.8.

This method for determining shear stresses in the webs of wide-flange beams also can be used for other sections having thin webs. For instance, Example 5-15 illustrates the procedure for a T-beam.

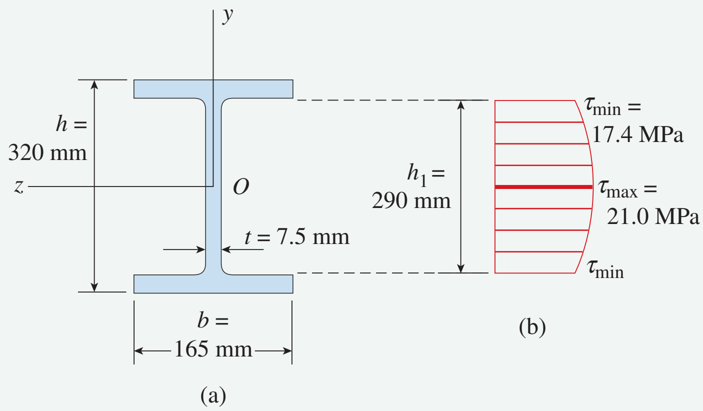
Example 5-14

A beam of wide-flange shape (Fig. 5-41a) is subjected to a vertical shear force $V = 45$ kN. The cross-sectional dimensions of the beam are $b = 165$ mm, $t = 7.5$ mm, $h = 320$ mm, and $h_1 = 290$ mm.

Determine the maximum shear stress, minimum shear stress, and total shear force in the web. (Disregard the areas of the fillets when making calculations.)

FIGURE 5-41

Example 5-14: Shear stresses in the web of a wide-flange beam



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. *Conceptualize, Categorize*:

Maximum and minimum shear stresses: The maximum and minimum shear stresses in the web of the beam are given by Eqs. (5-53a and b). Before substituting into those equations, calculate the moment of inertia of the cross-sectional area from Eq. (5-52):

$$I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3) = 130.45 \times 10^6 \text{ mm}^4$$

3. ***Analyze***: Now substitute this value for I, as well as the numerical values for the shear force V and the cross-sectional dimensions, into Eqs. (5-53a and b):

$$\tau_{max} = \frac{V}{8It}(bh^2 - bh_1^2 + th_1^2) = 21.0 \text{ MPa}$$

$$\tau_{min} = \frac{Vb}{8It}(h^2 - h_1^2) = 17.4 \text{ MPa}$$

In this case, the ratio of τ_{max} to τ_{min} is 1.21, that is, the maximum stress in the web is 21% larger than the minimum stress. The variation of the shear stresses over the height h_1 of the web is shown in Fig. 5-41b.

Total shear force: The shear force in the web is calculated from Eq. (5-54) as

$$V_{web} = \frac{th_1}{3}(2\tau_{max} + \tau_{min}) = 43.0 \text{ kN}$$

4. ***Finalize***: From this result, note that the web of this particular beam resists 96% of the total shear force.

Note: The average shear stress in the web of the beam [from Eq. (5-55)] is

$$\tau_{aver} = \frac{V}{th_1} = 20.7 \text{ MPa}$$

which is only 1% less than the maximum stress.

Example 5-15

A beam having a T-shaped cross section (Fig. 5-42a) is subjected to a vertical shear force $V = 10{,}000$ lb. The cross-sectional dimensions are $b = 4$ in., $t = 1.0$ in., $h = 8.0$ in., and $h_1 = 7.0$ in.

Determine the shear stress τ_1 at the top of the web (level nn) and the maximum shear stress τ_{max}. (Disregard the areas of the fillets.)

Solution:

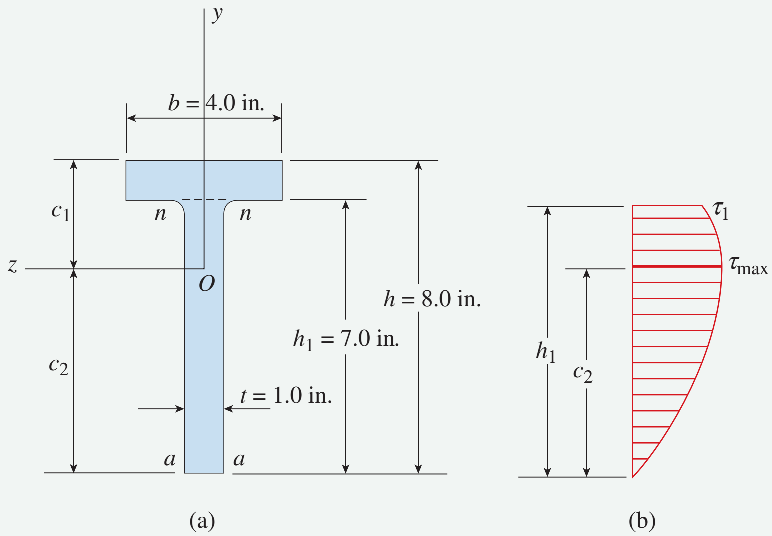
Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. ***Conceptualize, Categorize***:

Location of neutral axis: The neutral axis of the T-beam is located by calculating the distances c_1 and c_2 from the top and bottom of the beam to the centroid of the cross section (Fig. 5-42a). First, divide the cross section into two rectangles: the flange and the web (see the dashed line in Fig. 5-42a). Then calculate the first moment Q_{aa} of these two rectangles with respect to line aa at the bottom of the beam. The distance c_2 is equal to Q_{aa} divided by the area

FIGURE 5-42

Example 5-15: Shear stresses in web of T-shaped beam



A of the entire cross section (see Appendix D, Section D.2, for methods for locating centroids of composite areas). The calculations are

$$A = \Sigma A_i = b(h - h_1) + th_1 = 11.0 \text{ in}^2$$

$$Q_{aa} = \Sigma y_i A_i = \left(\frac{h + h_1}{2}\right)(b)(h - h_1) + \frac{h_1}{2}(th_1) = 54.5 \text{ in}^3$$

$$c_2 = \frac{Q_{aa}}{A} = \frac{54.5 \text{ in}^3}{11.0 \text{ in}^2} = 4.955 \text{ in.} \quad c_1 = h - c_2 = 3.045 \text{ in.}$$

Moment of inertia: Find the moment of inertia I of the entire cross-sectional area (with respect to the neutral axis) by determining the moment of inertia I_{aa} about line aa at the bottom of the beam and then use the parallel-axis theorem (see Section D.4, Appendix D):

$$I = I_{aa} - Ac_2^2$$

The calculations are

$$I_{aa} = \frac{bh^3}{3} - \frac{(b - t)h_1^3}{3} = 339.67 \text{ in}^4 \quad Ac_2^2 = 270.02 \text{ in}^4 \quad I = 69.65 \text{ in}^4$$

3. ***Analyze*:**

Shear stress at top of web: To find the shear stress τ_1 at the top of the web (along line nn) calculate the first moment Q_1 of the area above level nn. This first moment is equal to the area of the flange times the distance from the neutral axis to the centroid of the flange:

$$Q_1 = b(h - h_1)\left(c_1 - \frac{h - h_1}{2}\right)$$
$$= (4 \text{ in.})(1 \text{ in.})(3.045 \text{ in.} - 0.5 \text{ in.}) = 10.18 \text{ in}^3$$

You get the same result if you calculate the first moment of the area below level *nn*:

$$Q_1 = th_1\left(c_2 - \frac{h_1}{2}\right) = (1\text{ in.})(7\text{ in.})(4.955\text{ in.} - 3.5\text{ in.}) = 10.18\text{ in}^3$$

Substitute into the shear formula to find

$$\tau_1 = \frac{VQ_1}{It} = \frac{(10{,}000\text{ lb})(10.18\text{ in}^3)}{(69.65\text{ in}^4)(1\text{ in.})} = 1460\text{ psi}$$

This stress exists both as a vertical shear stress acting on the cross section and as a horizontal shear stress acting on the horizontal plane between the flange and the web.

Maximum shear stress: The maximum shear stress occurs in the web at the neutral axis. Therefore, calculate the first moment Q_{max} of the cross-sectional area below the neutral axis:

$$Q_{max} = tc_2\left(\frac{c_2}{2}\right) = (1\text{ in.})(4.955\text{ in.})\left(\frac{4.955\text{ in.}}{2}\right) = 12.28\text{ in}^3$$

The same result is obtained if the first moment of the area above the neutral axis is computed, but those calculations would be slighter longer.

Substitute into the shear formula to obtain

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{(10{,}000\text{ lb})(12.28\text{ in}^3)}{(69.65\text{ in}^4)(1\text{ in.})} = 1760\text{ psi}$$

which is the maximum shear stress in the beam.

4. ***Finalize***: The parabolic distribution of shear stresses in the web is shown in Fig. 5-42b.

*5.11 Built-Up Beams and Shear Flow

Built-up beams are fabricated from two or more pieces of material joined together to form a single beam. Such beams can be constructed in a great variety of shapes to meet special architectural or structural needs and to provide larger cross sections than are ordinarily available.

Figure 5-43 shows some typical cross sections of built-up beams. A wood **box beam** (Fig. 5-43a) is constructed of two planks that serve as flanges and two plywood webs. The pieces are joined together with nails, screws, or glue in such a manner that the entire beam acts as a single unit. Box beams are also constructed of other materials, including steel, plastics, and composites.

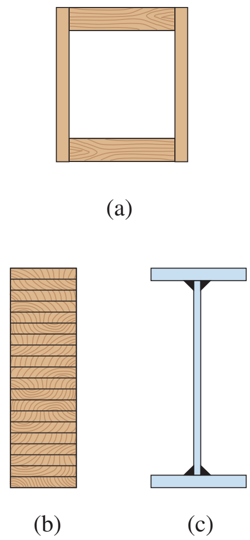
The second example (Fig. 5-43b) is a glued laminated beam (called a **glulam beam**) made of boards glued together to form a much larger beam than could be cut from a tree as a single member. Glulam beams are widely used in the construction of small buildings.

The third example (Fig. 5-43c) is a steel **plate girder** of the type commonly used in bridges and large buildings. These girders, consisting of three steel plates joined by welding, can be fabricated in much larger sizes than are available with ordinary wide-flange or I-beams.

Built-up beams must be designed so that the beam behaves as a single member. Consequently, the design calculations involve two phases. In the first phase, the beam is designed as though it were made of one piece, taking into account both bending and shear stresses. In the second phase, the *connections* between the parts (such as nails, bolts, welds, and glue) are designed to ensure that the beam does indeed behave as a single entity. In particular, the connections must be strong enough to transmit the horizontal shear forces acting between the parts of the beam. To obtain these forces, make use of the concept of *shear flow*.

FIGURE 5-43

Cross sections of typical built-up beams: (a) wood box beam, (b) glulam beam, and (c) plate girder



Shear Flow

To obtain a formula for the horizontal shear forces acting between parts of a beam, return to the derivation of the shear formula (see Figs. 5-30 and 5-31 of Section 5.8). In that derivation, element mm_1n_1n was cut from a beam (Fig. 5-44a) and horizontal equilibrium of a subelement mm_1p_1p was investigated (Fig. 5-44b). From the horizontal equilibrium of the subelement, the force F_3 (Fig. 5-44c) acting on its lower surface was found to be

$$F_3 = \frac{dM}{I}\int y\,dA \qquad \textbf{(5-56)}$$

This equation is repeated from Eq. (5-36) of Section 5.8.

FIGURE 5-44

Horizontal shear stresses and shear forces in a beam (*Note*: These figures are repeated from Figs. 5-30 and 5-31)

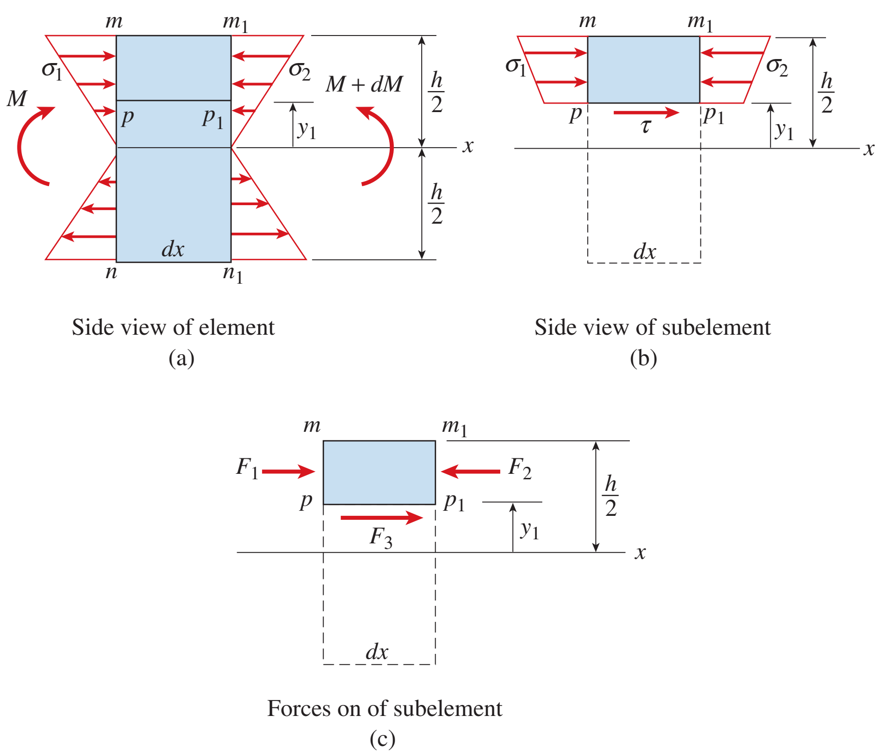
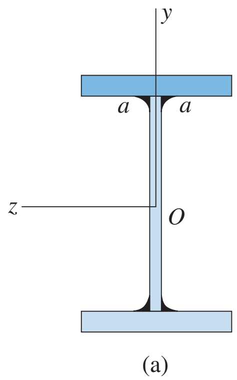
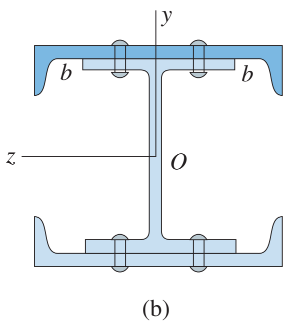
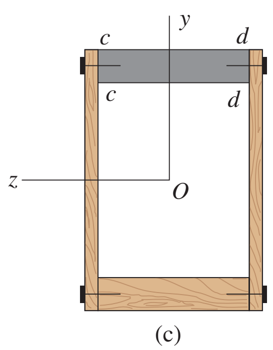


FIGURE 5-45

Areas used when calculating the first moment Q







Now define a new quantity called the **shear flow** f. Shear flow is the *horizontal shear force per unit distance along the longitudinal axis of the beam*. Since the force F_3 acts along the distance dx, the shear force per unit distance is equal to F_3 divided by dx; thus,

$$f = \frac{F_3}{dx} = \frac{dM}{dx}\left(\frac{1}{I}\right)\int y\,dA$$

Replacing dM/dx by the shear force V and denoting the integral by Q leads to the **shear-flow formula**:

$$f = \frac{VQ}{I} \quad \textbf{(5-57)}$$

This equation gives the shear flow acting on the horizontal plane pp_1 shown in Fig. 5-44a. The terms V, Q, and I have the same meanings as in the shear formula [Eq. (5-41)].

If the shear stresses on plane pp_1 are uniformly distributed, as assumed for rectangular beams and wide-flange beams, the shear flow f equals τb. In that case, the shear-flow formula reduces to the shear formula. However, the derivation of Eq. (5-56) for the force F_3 does not involve any assumption about the distribution of shear stresses in the beam. Instead, the force F_3 is found solely from the horizontal equilibrium of the subelement (Fig. 5-44c). Therefore, the subelement and the force F_3 can be interpreted in more general terms than before.

The subelement may be *any* prismatic block of material between cross sections mn and m_1n_1 (Fig. 5-44a). It does not have to be obtained by making a single horizontal cut (such as pp_1) through the beam. Also, since the force F_3 is the total horizontal shear force acting between the subelement and the rest of the beam, it may be distributed anywhere over the sides of the subelement, not just on its lower surface. These same comments apply to the shear flow f, since it is merely the force F_3 per unit distance.

Now return to the shear-flow formula $f = VQ/I$ [Eq. (5-57)]. The terms V and I have their usual meanings and are not affected by the choice of subelement. However, the first moment Q is a property of the cross-sectional face of the subelement. To illustrate how Q is determined, consider three specific examples of built-up beams (Fig. 5-45).

Areas Used when Calculating the First Moment Q

The first example of a built-up beam is a welded steel **plate girder** (Fig. 5-45a). The welds must transmit the horizontal shear forces that act between the flanges and the web. At the upper flange, the horizontal shear force (per unit distance along the axis of the beam) is the shear flow along the contact surface aa. This shear flow may be calculated by taking Q as the first moment of the cross-sectional area above the contact surface aa. In other words, Q is the first moment of the flange area (shown shaded in Fig. 5-45a) calculated with respect to the neutral axis. After calculating the shear flow, next determine the amount of welding needed to resist the shear force, because the strength of a weld is usually specified in terms of force per unit distance along the weld.

The second example is a **wide-flange beam** that is strengthened by riveting a channel section to each flange (Fig. 5-45b). The horizontal shear force acting between each channel and the main beam must be transmitted by the rivets. This force is calculated from the shear-flow formula using Q as the first moment of the area of the entire channel (shown shaded in the figure). The resulting shear flow is the longitudinal force per unit distance acting along the contact surface *bb*, and the rivets must be of adequate size and longitudinal spacing to resist this force.

The last example is a **wood box beam** with two flanges and two webs that are connected by nails or screws (Fig. 5-45c). The total horizontal shear force between the upper flange and the webs is the shear flow acting along *both* contact surfaces *cc* and *dd*, and therefore the first moment Q is calculated for the upper flange (the shaded area). In other words, the shear flow calculated from the formula $f = VQ/I$ is the total shear flow along all contact surfaces that surround the area for which Q is computed. In this case, the shear flow f is resisted by the combined action of the nails on *both* sides of the beam, that is, at both *cc* and *dd*, as illustrated in the following example.

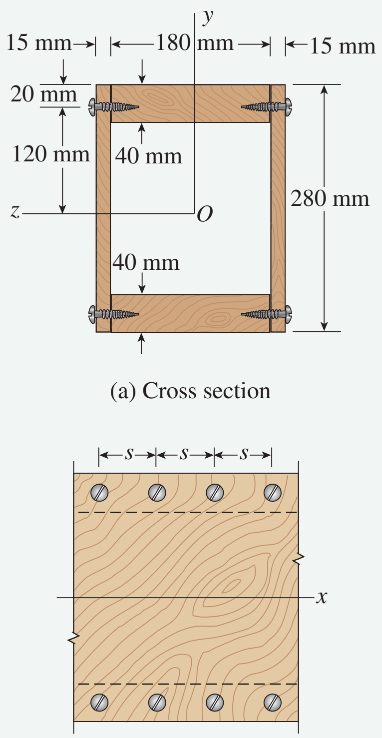
Example 5-16

A wood box beam (Fig. 5-46) is constructed of two boards, each 40 × 180 mm in cross section, that serve as flanges and two plywood webs, each 15 mm thick. The total height of the beam is 280 mm. The plywood is fastened to the flanges by wood screws having an allowable load in shear of $F = 800$ N each.

If the shear force V acting on the cross section is 10.5 kN, determine the maximum permissible longitudinal spacing s of the screws (Fig. 5-46b).

FIGURE 5-46

Example 5-16: Wood box beam



(a) Cross section

(b) Side view

Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

1, 2. *Conceptualize, Categorize*:

Shear flow: The horizontal shear force transmitted between the upper flange and the two webs can be found from the shear-flow formula $f = VQ/I$, in which Q is the first moment of the cross-sectional area of the flange. To find this first moment, multiply the area A_f of the flange by the distance d_f from its centroid to the neutral axis:

$$A_f = 40 \text{ mm} \times 180 \text{ mm} = 7200 \text{ mm}^2 \qquad d_f = 120 \text{ mm}$$

$$Q = A_f d_f = (7200 \text{ mm}^2)(120 \text{ mm}) = 864 \times 10^3 \text{ mm}$$

The moment of inertia of the entire cross-sectional area about the neutral axis is equal to the moment of inertia of the outer rectangle minus the moment of inertia of the "hole" (the inner rectangle):

$$I = \frac{1}{12}(210 \text{ mm})(280 \text{ mm})^3 - \frac{1}{12}(180 \text{ mm})(200 \text{ mm})^3$$
$$= 264.2 \times 10^6 \text{ mm}^4$$

Substituting V, Q, and I into the shear-flow formula [Eq. (5-57)] gives

$$f = \frac{VQ}{I} = \frac{(10{,}500 \text{ N})(864 \times 10^3 \text{ mm}^3)}{264.2 \times 10^6 \text{ mm}^4} = 34.3 \text{ N/mm}$$

which is the horizontal shear force per millimeter of length that must be transmitted between the flange and the two webs.

3. ***Analyze***:

Spacing of screws: Since the longitudinal spacing of the screws is s, and since there are two lines of screws (one on each side of the flange), the load capacity of the screws is $2F$ per distance s along the beam. Therefore, the capacity of the screws per unit distance along the beam is $2F/s$. Equating $2F/s$ to the shear flow f and solving for the spacing s gives

$$s = \frac{2F}{f} = \frac{2(800 \text{ N})}{34.3 \text{ N/mm}} = 46.6 \text{ mm}$$

4. ***Finalize***: This value of s is the maximum permissible spacing of the screws based upon the allowable load per screw. Any spacing greater than 46.6 mm would overload the screws. For convenience in fabrication (and to be on the safe side), a spacing such as $s = 45$ mm should be selected.

*5.12 Beams with Axial Loads

Structural members are often subjected to the simultaneous action of bending loads and axial loads. This happens, for instance, in aircraft frames, columns in buildings, machinery, parts of ships, and spacecraft. If the members are not too slender, the combined stresses can be obtained by superposition of the bending stresses and the axial stresses.

To see how this is accomplished, consider the cantilever beam shown in Fig. 5-47a. The only load on the beam is an inclined force P acting through the centroid of the end cross section. This load can be resolved into two components, a lateral load Q and an axial load S. These loads produce **stress resultants** in the form of bending moments M, shear forces V, and axial forces N throughout the beam (Fig. 5-47b). On a typical cross section a distance x from the support, these stress resultants are

$$M = Q(L - x) \quad V = -Q \quad N = S$$

in which L is the length of the beam. The stresses associated with each of these stress resultants can be determined at any point in the cross section by means of the appropriate formula ($\sigma = -My/I$, $\tau = VQ/Ib$, and $\sigma = N/A$).

Since both the axial force N and bending moment M produce normal stresses, combine those stresses to obtain the final stress distribution. The **axial force** (when acting alone) produces a uniform stress distribution $\sigma = N/A$

over the entire cross section, as shown by the stress diagram in Fig. 5-47c. In this particular example, the stress σ is tensile, as indicated by the plus signs attached to the diagram.

The **bending moment** produces a linearly varying stress $\sigma = -My/I$ (Fig. 5-47d) with compression on the upper part of the beam and tension on the lower part. The distance y is measured from the z axis, which passes through the centroid of the cross section.

The final distribution of normal stresses is obtained by superposing the stresses produced by the axial force and the bending moment. Thus, the equation for the **combined stresses** is

$$\sigma = \frac{N}{A} - \frac{My}{I} \qquad \textbf{(5-58)}$$

Note that N is positive when it produces tension and M is positive, according to the bending-moment sign convention (positive bending moment produces compression in the upper part of the beam and tension in the lower part). Also, the y axis is positive upward. As long as these sign conventions are used in Eq. (5-58), the normal stress σ is positive for tension and negative for compression.

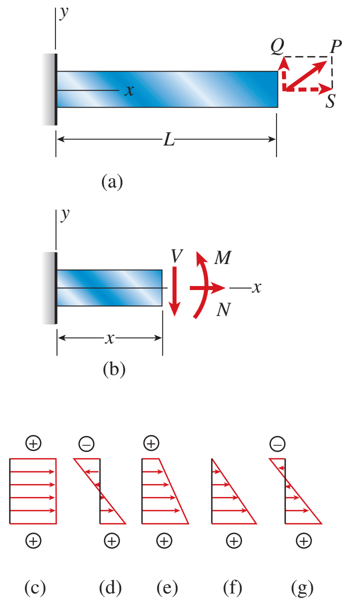
The final stress distribution depends upon the relative algebraic values of the terms in Eq. (5-58). For this example, the three possibilities are shown in Figs. 5-47e, f, and g. If the bending stress at the top of the beam (Fig. 5-47d) is numerically less than the axial stress (Fig. 5-47c), the entire cross section is in tension, as shown in Fig. 5-47e. If the bending stress at the top equals the axial stress, the distribution is triangular (Fig. 5-47f), and if the bending stress is numerically larger than the axial stress, the cross section is partially in compression and partially in tension (Fig. 5-47g). Of course, if the axial force is a compressive force, or if the bending moment is reversed in direction, the stress distributions change accordingly.

Whenever bending and axial loads act simultaneously, the neutral axis (that is, the line in the cross section where the normal stress is zero) no longer passes through the centroid of the cross section. As shown in Figs. 5-47e, f, and g, respectively, the neutral axis may be outside the cross section, at the edge of the section, or within the section.

The use of Eq. (5-58) to determine the stresses in a beam with axial loads is illustrated later in Example 5-17.

FIGURE 5-47

Normal stresses in a cantilever beam subjected to both bending and axial loads: (a) beam with load P acting at the free end, (b) stress resultants N, V, and M acting on a cross section at distance x from the support, (c) tensile stresses due to the axial force N acting alone, (d) tensile and compressive stresses due to the bending moment M acting alone, and (e), (f), (g) are possible final stress distributions due to the combined effects of N and M



Eccentric Axial Loads

An **eccentric axial load** is an axial force that does *not* act through the centroid of the cross section. An example is shown in Fig. 5-48a, where the cantilever beam AB is subjected to a tensile load P acting at distance e from the x axis (the x axis passes through the centroids of the cross sections). The distance e, called the *eccentricity* of the load, is positive in the positive direction of the y axis.

The eccentric load P is statically equivalent to an axial force P acting along the x axis and a bending moment Pe acting about the z axis (Fig. 5-48b). Note that the moment Pe is a negative bending moment.

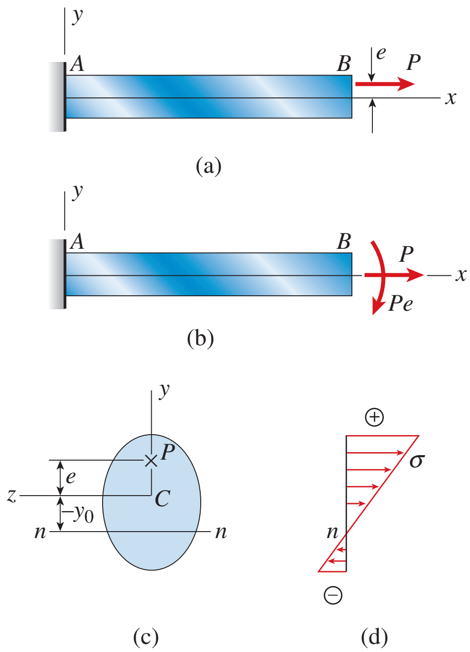
A cross-sectional view of the beam (Fig. 5-48c) shows the y and z axes passing through the centroid C of the cross section. The eccentric load P intersects the y axis, which is an axis of symmetry.

(Lester Lefkowitz/Getty Images)

Bending due to self-weight of beam and axial compression due to horizontal component of cable lifting force

FIGURE 5-48

(a) Cantilever beam with an eccentric axial load P,
(b) equivalent loads P and Pe,
(c) cross section of beam, and
(d) distribution of normal stresses over the cross section



Since the axial force N at any cross section is equal to P, and since the bending moment M is equal to $-Pe$, the **normal stress** at any point in the cross section [from Eq. (5-58)] is

$$\sigma = \frac{P}{A} + \frac{Pey}{I} \tag{5-59}$$

in which A is the area of the cross section and I is the moment of inertia about the z axis. The stress distribution obtained from Eq. (5-59), for the case where both P and e are positive, is shown in Fig. 5-48d.

The position of the **neutral axis** nn (Fig. 5-48c) can be obtained from Eq. (5-59) by setting the stress σ equal to zero and solving for the coordinate y, denoted as y_0. The result is

$$y_0 = -\frac{I}{Ae} \tag{5-60}$$

The coordinate y_0 is measured from the z axis (which is the neutral axis under pure bending) to the line nn of zero stress (the neutral axis under combined bending and axial load). Because y_0 is positive in the direction of the y axis (upward in Fig. 5-48c), it is labeled $-y_0$ when it is shown downward in the figure.

From Eq. (5-60), note that the neutral axis lies below the z axis when e is positive and above the z axis when e is negative. If the eccentricity is reduced, the distance y_0 increases and the neutral axis moves away from the centroid. In the limit, as e approaches zero, the load acts at the centroid, the neutral axis is at an infinite distance, and the stress distribution is uniform. If the eccentricity is increased, the distance y_0 decreases and the neutral axis moves toward the centroid. In the limit, as e becomes extremely large, the load acts at an infinite distance, the neutral axis passes through the centroid, and the stress distribution is the same as in pure bending.

Eccentric axial loads are analyzed in some of the problems at the end of this chapter.

Limitations

The preceding analysis of beams with axial loads is based upon the assumption that the bending moments can be calculated without considering the deflections of the beams. In other words, when determining the bending moment M for use in Eq. (5-58), you must be able to use the original dimensions of the beam—that is, the dimensions *before* any deformations or deflections occur. The use of the original dimensions is valid provided the beams are relatively stiff in bending, so that the deflections are very small.

Thus, when analyzing a beam with axial loads, it is important to distinguish between a **stocky beam**, which is relatively short and therefore highly resistant to bending, and a **slender beam**, which is relatively long and therefore very flexible. In the case of a stocky beam, the lateral deflections are so small as to have no significant effect on the line of action of the axial forces. As a consequence, the bending moments will not depend upon the deflections, and the stresses can be found from Eq. (5-58).

In the case of a slender beam, the lateral deflections (even though small in magnitude) are large enough to alter significantly the line of action of the axial forces. When that happens, an additional bending moment equal to the product of the axial force and the lateral deflection is created at every cross section. In other words, there is an interaction, or coupling, between the axial effects and the bending effects. This type of behavior is discussed in Chapter 11 on **columns**.

The distinction between a stocky beam and a slender beam is obviously not a precise one. In general, the only way to know whether interaction effects are important is to analyze the beam with and without the interaction and notice whether the results differ significantly. However, this procedure may require considerable calculating effort. Therefore, as a guideline for practical use, consider a beam with a length-to-height ratio of 10 or less to be a stocky beam. Only stocky beams are considered in the problems pertaining to this section.

Example 5-17

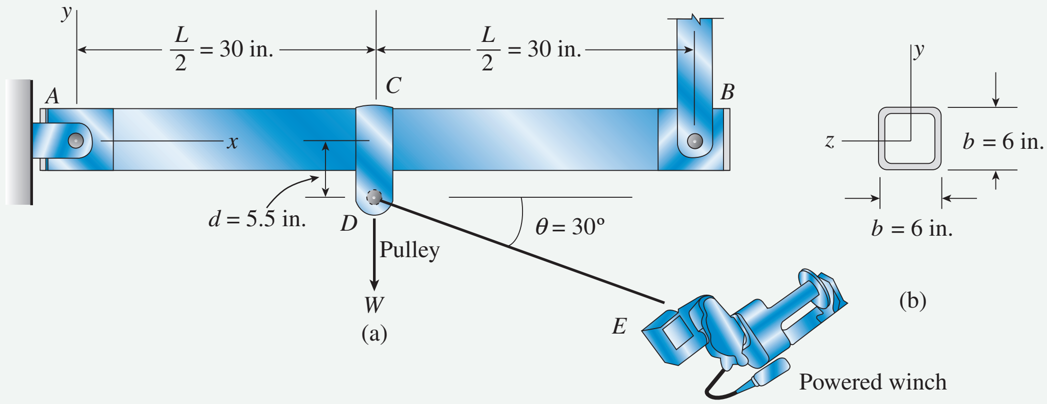
A tubular beam ACB with a length of $L = 60$ in. is pin-supported at its ends, A and B. A powered winch at E lifts load W below C using a cable which passes over a frictionless pulley at midlength (point D, Fig. 5-49a). The distance from the center of the pulley to the longitudinal axis of the tube is $d = 5.5$ in. The cross section of the tube is square (Fig. 5-49b) with an outer dimension of $b = 6.0$ in., area of $A = 20.0\ \text{in}^2$, and moment of inertia of $I = 86.67\ \text{in}^4$.

(a) Determine the maximum tensile and compressive stresses in the beam due to a load $W = 3000$ lb.

(b) If the allowable normal stress in the tube is 3500 psi, find the maximum permissible load W. Assume that the cable, pulley, and bracket CD are adequate to carry load W_{max}.

FIGURE 5-49

Example 5-17: Tubular beam subjected to combined bending and axial load



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

Part (a): Maximum tensile and compressive stresses in the beam.

1, 2. ***Conceptualize, Categorize:***

Beam and loading: Begin by representing the beam and its load in idealized form for the purposes of analysis (Fig. 5-50a). Since the support at end A resists both horizontal and vertical displacement, it is represented as a pin support. The support at B prevents vertical displacement but offers no resistance to horizontal displacement, so it is shown as a roller support.

Replace the cable forces at D with statically equivalent forces F_H and F_V and moment M_O, all of which are applied on the axis of the beam at C (see Fig. 5-50a):

$$F_H = W\cos(\theta) = 2598 \text{ lb} \qquad F_V = W[1 + \sin(\theta)] = 4500 \text{ lb}$$
$$M_0 W\cos(\theta)d = 14{,}289 \text{ lb-in.}$$

FIGURE 5-50

Solution of Example 5-17:
(a) Idealized beam and loading,
(b) axial-force diagram,
(c) shear-force diagram,
and (d) bending-moment diagram

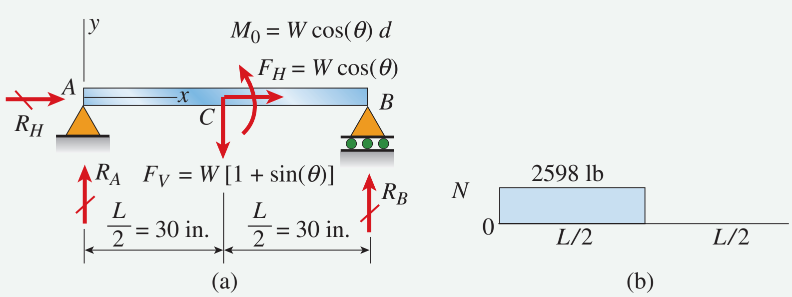
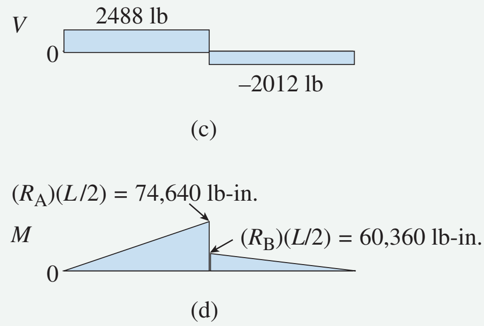


FIGURE 5-50 (Continued)

Solution of Example 5-17:
(a) Idealized beam and loading,
(b) axial-force diagram,
(c) shear-force diagram,
and (d) bending-moment
diagram



Reactions and stress resultants: The reactions of the beam (R_H, R_A, and R_B) are labeled in Fig. 5-50a. Also, the diagrams of axial force N, shear force V, and bending moment M are shown in Figs. 5-50b, c, and d, respectively. All of these quantities are found from free-body diagrams and equations of equilibrium using the techniques described in Chapter 4. For example, use equations of statics to find that

$$\Sigma F_H = 0: \quad R_H = -F_H = -W\cos(\theta) = -(3000\text{ lb})\cos(30°) = -2598\text{ lb} \qquad \textbf{(a)}$$

$$\Sigma M_A = 0: \quad R_B = \frac{1}{L}\left(F_V\frac{L}{2} - M_0\right) = \frac{W}{2}[1 + \sin(\theta)] - W\frac{d}{L}[\cos(\theta)]$$

$$R_B = (3000\text{ lb})\left[\frac{1 + \sin(30°)}{2} - \left(\frac{5.5\text{ in.}}{60\text{ in.}}\right)\cos(30°)\right] = 2012\text{ lb} \qquad \textbf{(b)}$$

$$\Sigma F_V = 0: \quad R_A = F_V - R_B = (3000\text{ lb})(1 + \sin(30°)) - 2012\text{ lb} = 2488\text{ lb} \qquad \textbf{(c)}$$

Next, use the axial-force (N), shear-force (V), and bending-moment (M) diagrams (Figs. 5-50b, c, and d, respectively) to find the combined stresses in beam ACB using Eq. (5-58).

3. ***Analyze*:**

Stresses in the beam: The *maximum tensile stress* in the beam occurs at the bottom of the beam ($y = -3.0$ in.) just to the left of the midpoint C. Note that at this point in the beam the tensile stress due to the axial force adds to the tensile stress produced by the largest bending moment. Thus, from Eq. (5-58),

$$(\sigma_t)_{\max} = \frac{N}{A} - \frac{My}{I} = \frac{2598\text{ lb}}{20\text{ in}^2} - \frac{(74{,}640\text{ lb-in.})(-3\text{ in.})}{86.67\text{ in}^4}$$
$$= 130\text{ psi} + 2583\text{ psi} = 2713\text{ psi}$$

The *maximum compressive stress* occurs either at the top of the beam ($y = 3.0$ in.) to the left of point C or at the top of the beam to the right of point C. These two stresses are calculated as

$$(\sigma_c)_{\text{left}} = \frac{N}{A} - \frac{My}{I} = \frac{2598 \text{ lb}}{20 \text{ in}^2} - \frac{(74{,}640 \text{ lb-in.})(3 \text{ in.})}{86.67 \text{ in}^4}$$
$$= 130 \text{ psi} - 2583 \text{ psi} = -2453 \text{ psi}$$

$$(\sigma_c)_{\text{right}} = \frac{N}{A} - \frac{My}{I} = 0 - \frac{(60{,}360 \text{ lb-in.})(3 \text{ in.})}{86.67 \text{ in}^4} = -2089 \text{ psi}$$

Thus, the maximum compressive stress is

$$(\sigma_c)_{\max} = -2453 \text{ psi}$$

and occurs at the top of the beam to the left of point C.

Part (b): Maximum permissible load W.

1, 2. ***Conceptualize, Categorize***: From Eq. (a), the *tensile stress at the bottom of the beam* just left of C (equal to 2713 psi for a load $W = 3000$ lb) will reach allowable normal stress $\sigma_a = 3500$ psi first and thus will be the determining factor in finding $W_{\max}$. Using expressions for reactions [Eqs. (a), (b), and (c)], the axial tension force in beam segment AC and the positive moment just left of C are

$$N = W\cos(\theta) \qquad M = R_A\frac{L}{2} = W\left(\frac{1+\sin(\theta)}{2} + \frac{d}{L}\cos(\theta)\right)\left(\frac{L}{2}\right)$$

From Eq. (5-58), the combined normal stress is

$$\sigma_a = \frac{W\cos(\theta)}{A} - \frac{W\left(\frac{1+\sin(\theta)}{2} + \frac{d}{L}\cos(\theta)\right)\left(\frac{L}{2}\right)\left(\frac{-b}{2}\right)}{I}$$

3. ***Analyze***: Solving for $W = W_{\max}$ gives

$$W_{\max} = \frac{\sigma_a}{\dfrac{\cos(\theta)}{A} + \dfrac{bL[1+\sin(\theta)]}{8I} + \dfrac{bd\cos(\theta)}{4I}} = 3869 \text{ lb}$$

4. ***Finalize***: This example shows how the normal stresses in a beam due to combined bending and axial load can be determined. The shear stresses acting on cross sections of the beam (due to the shear forces V) can be determined independently of the normal stresses, as described earlier in this chapter. Later, in Chapter 7, stresses on inclined planes are computed when both the normal and shear stresses acting on cross-sectional planes are known.

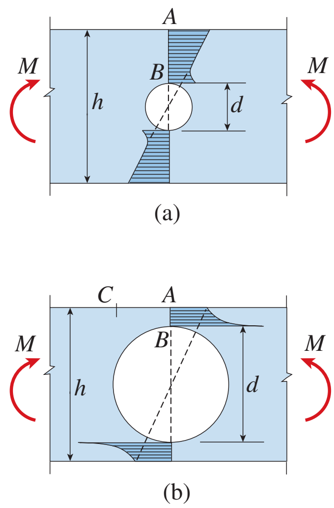
*5.13 Stress Concentrations in Bending

The flexure and shear formulas discussed in earlier sections of this chapter are valid for beams without holes, notches, or other abrupt changes in dimensions. Whenever such discontinuities exist, high localized stresses are produced. These **stress concentrations** can be extremely important when a member is made of brittle material or is subjected to dynamic loads. (See Chapter 2, Section 2.10, for a discussion of the conditions under which stress concentrations are important.)

For illustrative purposes, two cases of stress concentrations in beams are described in this section. The first case is a beam of rectangular cross section with a **hole at the neutral axis** (Fig. 5-51). The beam has a height h and thickness b (perpendicular to the plane of the figure) and is in pure bending under the action of bending moments M.

FIGURE 5-51

Stress distributions in a beam in pure bending with a circular hole at the neutral axis (The beam has a rectangular cross section with height h and thickness b)



When the diameter d of the hole is small compared to the height h, the stress distribution on the cross section through the hole is approximately as shown by the diagram in Fig. 5-51a. At point B on the edge of the hole, the stress is much larger than the stress that would exist at that point if the hole were not present. (The dashed line in the figure shows the stress distribution with no hole.) However, moving toward the outer edges of the beam (toward point A), the stress distribution varies linearly with distance from the neutral axis and is only slightly affected by the presence of the hole.

When the hole is relatively large, the stress pattern is approximately as shown in Fig. 5-51b. There is a large increase in stress at point B and only a small change in stress at point A, as compared to the stress distribution in the beam without a hole (again shown by the dashed line). The stress at point C is larger than the stress at A but smaller than the stress at B.

Extensive investigations have shown that the stress at the edge of the hole (point B) is approximately twice the *nominal stress* at that point. The nominal stress is calculated from the flexure formula in the standard way, that is, $\sigma = My/I$, in which y is the distance $d/2$ from the neutral axis to point B and I is the moment of inertia of the net cross section at the hole. Thus, the following approximate formula can be used to find the stress at point B:

$$\sigma_B \approx 2\frac{My}{I} = \frac{12Md}{b(h^3 - d^3)} \tag{5-61}$$

At the outer edge of the beam (at point C), the stress is approximately equal to the *nominal stress* (not the actual stress) at point A (where $y = h/2$):

$$\sigma_C \approx \frac{My}{I} = \frac{6Mh}{b(h^3 - d^3)} \tag{5-62}$$

From the last two equations, the ratio σ_B/σ_C is approximately $2d/h$. Hence, when the ratio d/h of hole diameter to height of beam exceeds 1/2, the largest stress occurs at point B. When d/h is less than 1/2, the largest stress is at point C.

The second case is a **rectangular beam with notches** (Fig. 5-52). The beam shown in the figure is subjected to pure bending and has a height h and thickness b (perpendicular to the plane of the figure). Also, the net height of the beam (that is, the distance between the bases of the notches) is h_1, and the radius at the base of each notch is R. The maximum stress in this beam occurs at the base

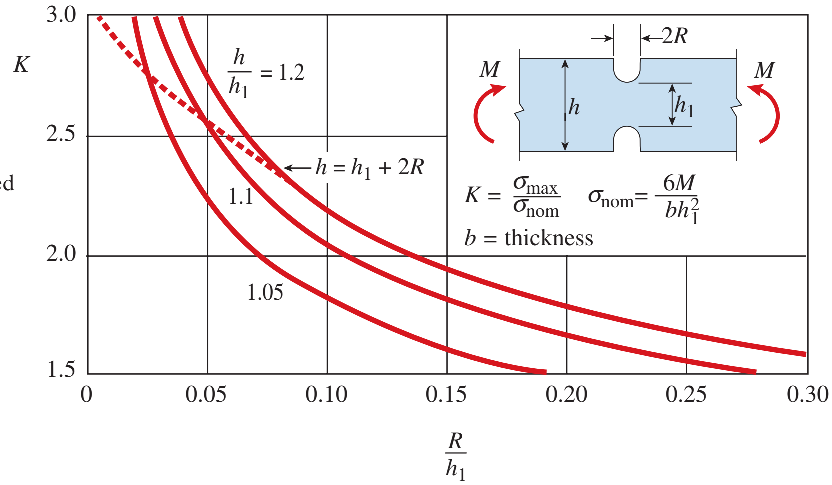


FIGURE 5-52

Stress-concentration factor K for a notched beam of rectangular cross section in pure bending (h = height of beam; b = thickness of beam, perpendicular to the plane of the figure), where the dashed line is for semicircular notches ($h = h_1 + 2R$)

of the notches and may be much larger than the nominal stress at that same point. The nominal stress is calculated from the flexure formula with $y = h_1/2$ and $I = bh_1^3/12$; thus,

$$\sigma_{nom} = \frac{My}{I} = \frac{6M}{bh_1^2} \tag{5-63}$$

The maximum stress is equal to the stress-concentration factor K times the nominal stress:

$$\sigma_{max} = K\sigma_{nom} \tag{5-64}$$

The stress-concentration factor K is plotted in Fig. 5-52 for a few values of the ratio h/h_1. Note that when the notch becomes "sharper," that is, the ratio R/h_1 becomes smaller, the stress-concentration factor increases. (Fig. 5-52 is plotted from the formulas given in Ref. 2-9.)

The effects of the stress concentrations are confined to small regions around the holes and notches, as explained in the discussion of Saint-Venant's principle in Section 2.10. At a distance equal to h or greater from the hole or notch, the stress-concentration effect is negligible and the ordinary formulas for stresses may be used.

Example 5-18

A simple beam AB with rectangular cross section ($b \times h$) has a hole with a diameter of d at its centerline and two notches on either side and equidistant from the beam centerline. Beam AB is simply supported, and loads P are applied at $L/5$ from each end of the beam. Assume that dimensions given in Fig. 5-53 are $L = 4.5$ m, $b = 50$ mm, $h = 144$ mm, $h_1 = 120$ mm, $d = 85$ mm, and $R = 10$ mm. Assume that the allowable bending stress is $\sigma_a = 150$ MPa.

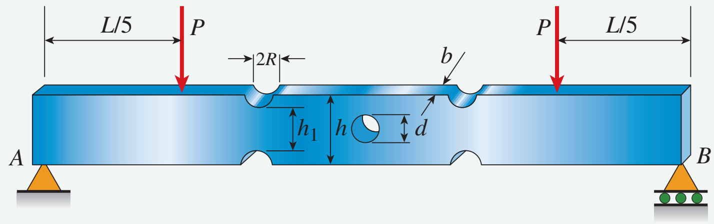
(a) Find the maximum permissible value of applied load P.

(b) If $P = 11$ kN, find the smallest acceptable radius of the notches, R_{min}.

(c) If $P = 11$ kN, find the maximum acceptable diameter of the hole at mid-height of beam.

FIGURE 5-53

Example 5-18: Rectangular steel beam with notches and a hole



Solution:

Use a four-step problem-solving approach. Combine steps as needed for an efficient solution.

Part (a): Maximum permissible load *P*.

1, 2. ***Conceptualize, Categorize*:** The central part of the beam between the loads P ($x = L/5$ to $x = 4L/5$) is in pure bending, and the maximum moment in this region is $M = PL/5$. To find P_{max}, compare the maximum bending stress (at midspan around the hole and in the notch regions) to the allowable stress value of $\sigma_a = 150$ MPa.

First, check the *maximum stresses around the hole*. The hole diameter-to-beam depth ratio d/h = 85 mm/144 mm = 0.59 exceeds 1/2, so the stress at B rather than at C (Fig. 5-51) will govern. Setting σ_B equal to σ_a and substituting $PL/5$ for M in Eq. (5-61) gives the expression for P_{max}:

$$M_{max} = \sigma_a\left[\frac{b(h^3 - d^3)}{12d}\right] \quad \text{and} \quad P_{max1} = \frac{5}{L}\left\{\sigma_a\left[\frac{b(h^3 - d^3)}{12d}\right]\right\}$$

3. ***Analyze*:** Use this expression to compute

$$P_{max1} = \frac{5}{4.5\text{ m}}\left\{150\text{ MPa}\left[\frac{50\text{ mm}[(144\text{ mm}^3) - (85\text{ mm})^3]}{12(85\text{ mm})}\right]\right\} = 19.38\text{ kN}$$

Next, check the peak stresses at the base of the two notches to get a second value of P_{max}. The ratio of notch radius R to height h_1 is equal to 0.083, and the ratio $h/h_1 = 1.2$. So from Fig. 5-52, the stress concentration factor K is approximately equal to 2.3 (see Fig. 5-54).

Use Eqs. (5-63) and (5-64) to get the expressions:

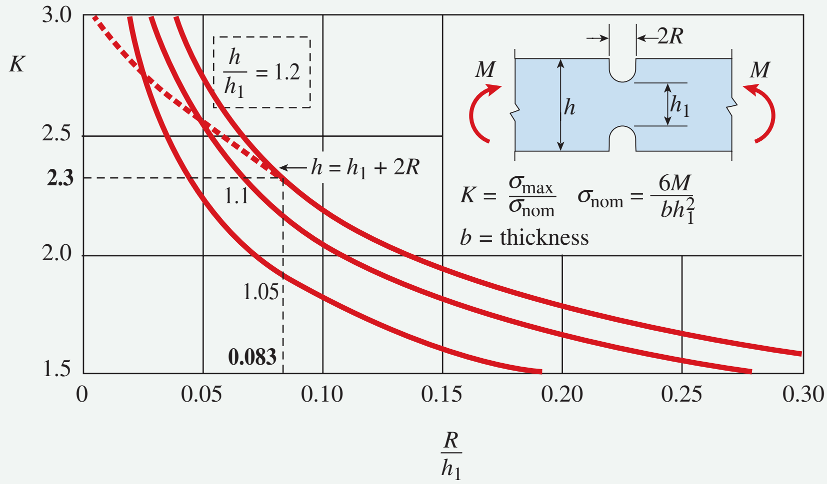
$$\sigma_{max} = K\sigma_{nom} = K\left(\frac{6M}{bh_1^2}\right) = K\left[\frac{6}{bh_1^2}\left(\frac{PL}{5}\right)\right]$$

so

$$P_{max2} = \sigma_a\left(\frac{5bh_1^2}{6KL}\right) = 150\text{ MPa}\left[\frac{5(50\text{ mm})(120\text{ mm})^2}{6(2.3)(4.5\text{ m})}\right] = 8.7\text{ kN}$$

FIGURE 5-54

Stress concentration factor K in notch regions of beam for part (a) of Example 5-18



4. ***Finalize***: Compare $P_{max\,1}$ and $P_{max\,2}$, to see that the peak stress at the base of the notches controls, so

$$P_{max} = 8.7\text{ kN}$$

Part (b): Smallest acceptable radius *R* of the notches.

1, 2. ***Conceptualize, Categorize***: The stress concentration factor K in Fig. 5-52 increases as the ratio of the notch radius R to dimension h_1 decreases.

3, 4. ***Analyze, Finalize***: Compute the nominal stress using Eq. (5-63) as

$$\sigma_{nom} = \frac{6\left(\frac{PL}{5}\right)}{bh_1^2} = \frac{6(11\text{ kN})(4.5\text{ m})}{5(50\text{ mm})(120\text{ mm})^2} = 82.5\text{ MPa}$$

Then set the maximum bending stress σ_{max} equal to the allowable stress $\sigma_a = 150$ MPa to find the stress concentration factor K:

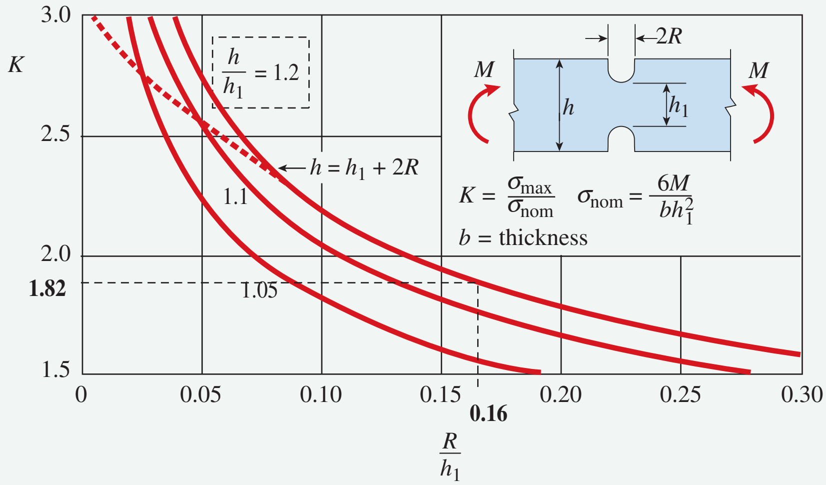
$$K = \frac{\sigma_a}{\sigma_{nom}} = \frac{150\text{ MPa}}{82.5\text{ MPa}} = 1.82$$

From Fig. 5-55, with $h/h_1 = 1.2$ and $K = 1.82$, obtain

$$\frac{R}{h_1} = 0.16 \quad \text{so} \quad R_{min} = 0.16(120\text{ mm}) = 19.2\text{ mm}$$

FIGURE 5-55

Stress concentration factor K in notch regions of beam for part (b) of Example 5-18



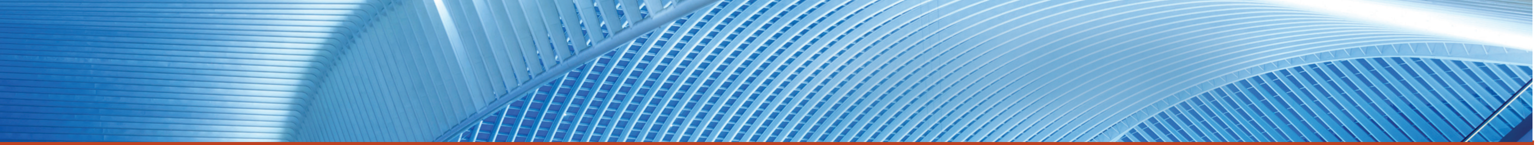
Part (c): Maximum acceptable diameter of the hole.

1, 2. ***Conceptualize, Categorize*:** Begin by assuming that ratio $d/h > 1/2$, and start with Eq. (5-61) (which assumes that maximum bending stress is at B, as in Fig. 5-51) to find d_{max}. If d/h turns out to be less than 1/2, use Eq. (5-62), which means that maximum bending stress is in fact at point C. If the peak stress is at B, write Eq. (5-61) as

$$\frac{12\left(\frac{PL}{5}\right)d}{b(h^3 - d^3)} = \sigma_a$$

3. ***Analyze*:** Solve the previous equation numerically to find that $d_{max} = 108.3$ mm.

4. ***Finalize*:** The original assumption about the d/h ratio is confirmed, since $d_{max}/h = 0.752$ exceeds 1/2, so the peak stress is indeed at B and not at C.



CHAPTER SUMMARY AND REVIEW

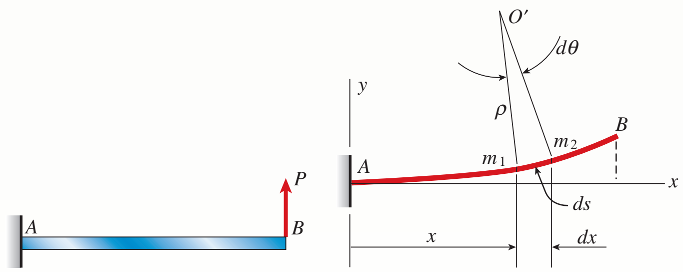
Chapter 5 covered the behavior of beams with loads applied and bending occurring in the x-y plane: a plane of symmetry in the beam cross section. Both pure bending and nonuniform bending were considered. The normal stresses (σ) were seen to vary linearly from the neutral surface in accordance with the **flexure formula**. Horizontal and vertical shear stresses (τ) were computed using the **shear formula** for the case of nonuniform bending of beams with either rectangular or circular cross sections. The special cases of shear in beams with flanges and built-up beams also were considered. Finally, stocky beams with both axial and transverse loads were discussed, followed by an evaluation of localized stresses in beams with abrupt changes in cross section around notches or holes.

Here are some of the major concepts and findings presented in this chapter.

1. If the xy plane is a plane of symmetry of a beam cross section and applied loads act in the x-y plane, the bending deflections occur in this same plane, known as the **plane of bending**.

2. A beam in pure bending has constant curvature κ, and a beam in nonuniform bending has varying curvature. Longitudinal strains (ε_x) in a bent beam are proportional to its curvature, and the strains in a beam in pure bending vary linearly with distance from the neutral surface, regardless of the shape of the stress-strain curve of the material, as

$$\varepsilon_x = -\kappa y$$

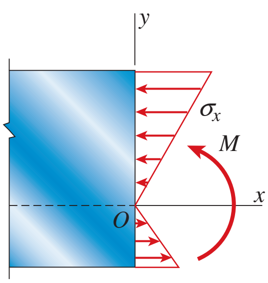


(a) (b)

3. The neutral axis passes through the centroid of the cross-sectional area when the material follows Hooke's law and there is no axial force acting on the cross section. When a beam of linearly elastic material is subjected to pure bending, the y and z axes are **principal centroidal** axes.

4. If the material of a beam is linearly elastic and follows Hooke's law, the **moment-curvature equation** shows that the curvature is directly proportional to the bending moment M and inversely proportional to the quantity EI, the **flexural rigidity** of the beam. The moment curvature relation is

$$\kappa = \frac{M}{EI}$$



5. The **flexure formula** shows that the normal stresses σ_x are directly proportional to the bending moment M and inversely proportional to the moment of inertia I of the cross section:

$$\sigma_x = -\frac{My}{I}$$

The maximum tensile and compressive bending stresses acting at any given cross section occur at points located farthest from the neutral axis. Thus,

$$(y = c_1,\ y = -c_2)$$

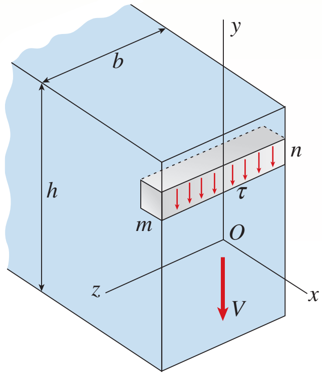
6. The normal stresses calculated from the flexure formula are not significantly altered by the presence of shear stresses and the associated warping of the cross section for the case of nonuniform bending. However, the flexure formula is not applicable near the supports of a beam or close to a concentrated load, because such irregularities produce **stress concentrations** that are much greater than the stresses obtained from the flexure formula.

7. To **design** a beam to resist bending stresses, calculate the required **section modulus** S from the maximum moment and allowable normal stress as

$$S = \frac{M_{\text{max}}}{\sigma_{\text{allow}}}$$

To minimize weight and save material, select a beam from a material design manual (see sample tables in Appendixes F and G for steel and wood) that has the least cross-sectional area while still providing the required section modulus; wide-flange sections and I-sections have most of their material in the flanges, and the width of their flanges helps to reduce the likelihood of sideways buckling.

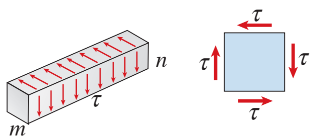
8. **Nonprismatic beams** (found in automobiles, airplanes, machinery, bridges, buildings, tools, and many other applications) commonly are used to reduce weight and improve appearance. The flexure formula gives reasonably accurate values for the bending stresses in nonprismatic beams, provided that the changes in cross-sectional dimensions are gradual. However, in a nonprismatic beam, the section modulus also varies along the axis, so do not assume that the maximum stresses occur at the cross section with the largest bending moment.



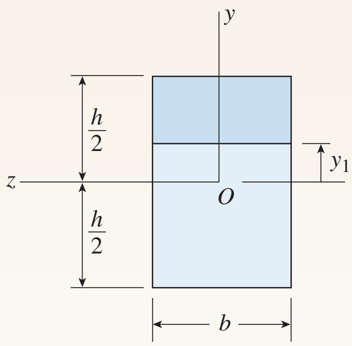
9. Beams subjected to loads that produce both bending moments (M) and shear forces (V) (**nonuniform bending**) develop both normal and shear stresses in the beam. Normal stresses are calculated from the **flexure formula** (provided the beam is constructed of a linearly elastic material), and shear stresses are computed using the **shear formula**

$$\tau = \frac{VQ}{Ib}$$

Shear stress varies parabolically over the height of a rectangular beam, and shear strain also varies parabolically; these shear strains cause cross sections of the beam that were originally plane surfaces to become warped. The maximum values of the shear stress and strain (τ_{max}, γ_{max}) occur at the neutral axis, and the shear stress and strain are zero on the top and bottom surfaces of the beam.



10. The shear formula applies only to prismatic beams and is valid only for beams of linearly elastic materials with small deflections; also, the edges of the cross section must be **parallel** to the y axis. For **rectangular** beams, the accuracy of the shear formula depends upon the height-to-width ratio of the cross section: The formula may be considered exact for very narrow beams but becomes less accurate as width b increases relative to height h.



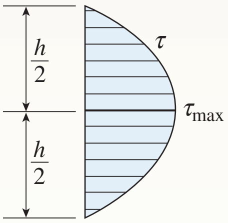
Use the shear formula to calculate the shear stresses only at the neutral axis of a beam of **circular** cross section.

For rectangular cross sections,

$$\tau_{\max} = \frac{3}{2}\frac{V}{A}$$

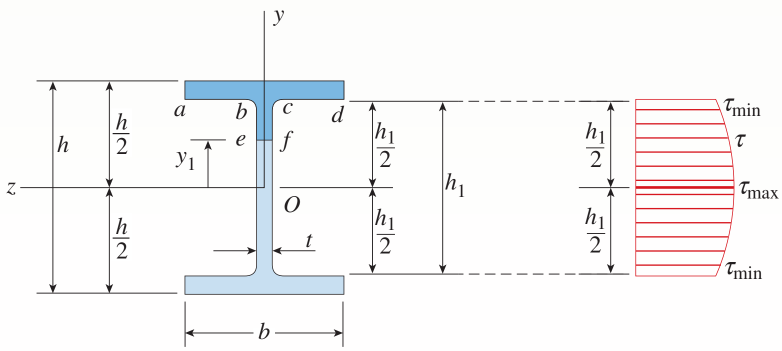
and for solid circular cross sections

$$\tau_{\max} = \frac{4}{3}\frac{V}{A}$$



11. Shear stresses rarely govern the design of either circular or rectangular beams made of metals such as steel and aluminum for which the allowable shear stress is usually in the range 25 to 50% of the allowable tensile stress. However, for **materials that are weak in shear**, such as wood, the allowable stress in horizontal shear is in the range of 4 to 10% of the allowable bending stress and so may govern the design.

12. Shear stresses in the flanges of **wide-flange beams** act in both vertical and horizontal directions. The horizontal shear stresses are much larger than the vertical shear stresses in the flanges. The shear stresses in the **web of a wide-flange beam** act only in the vertical direction, are larger than the stresses in the flanges, and may be computed using the shear formula. The maximum shear stress in the web of a wide-flange beam occurs at the neutral axis, and the minimum shear stress occurs where the web meets the flanges. For beams of typical proportions, the shear force in the web is 90 to 98% of the total shear force V acting on the cross section; the remainder is carried by shear in the flanges.



13. Connections between the parts in **built-up beams** (such as nails, bolts, welds, and glue) must be strong enough to transmit the horizontal shear forces acting between the parts of the beam. The connections are designed using the **shear flow formula**

$$f = \frac{VQ}{I}$$

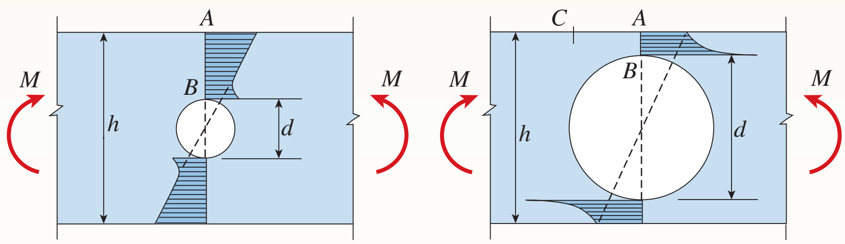
to ensure that the beam behaves as a single entity. **Shear flow** f is defined as horizontal shear force per unit distance along the longitudinal axis of the beam.

14. Normal stresses in **beams with axial loads** are obtained by superposing the stresses produced by the axial force N and the bending moment M as

$$\sigma = \frac{N}{A} - \frac{My}{I}$$

Whenever bending and axial loads act simultaneously, the neutral axis no longer passes through the centroid of the cross section and may be outside the cross section, at the edge of the section, or within the section. This discussion applies only to **stocky beams** for which the lateral deflections are so small as to have no significant effect on the line of action of the axial forces.

15. Stress distributions in beams are altered by holes, notches, or other abrupt changes in dimensions leading to high localized stresses or **stress concentrations**. These are especially important to consider when the beam material is brittle or the member is subjected to dynamic loads. The maximum stress values may be several times larger than the nominal stress.



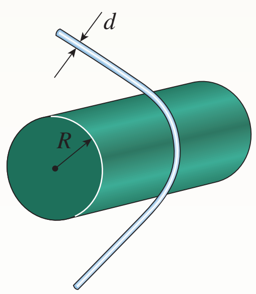
PROBLEMS Chapter 5

5.4 Longitudinal Strains in Beams

Introductory Problems

5.4-1 A steel wire with a diameter of $d = 1/16$ in. is bent around a cylindrical drum with a radius of $R = 36$ in. (see figure).

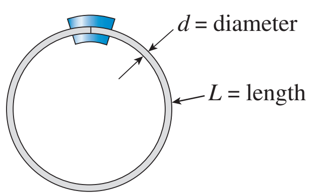
(a) Determine the maximum normal strain ε_{max}.
(b) What is the minimum acceptable radius of the drum if the maximum normal strain must remain below yield? Assume $E = 30{,}000$ ksi and $\sigma_Y = 100$ ksi.
(c) If $R = 36$ in., what is the maximum acceptable diameter of the wire if the maximum normal strain must remain below yield?



PROBLEM 5.4-1

5.4-2 A copper wire having a diameter of $d = 4$ mm is bent into a circle and held with the ends just touching (see figure).

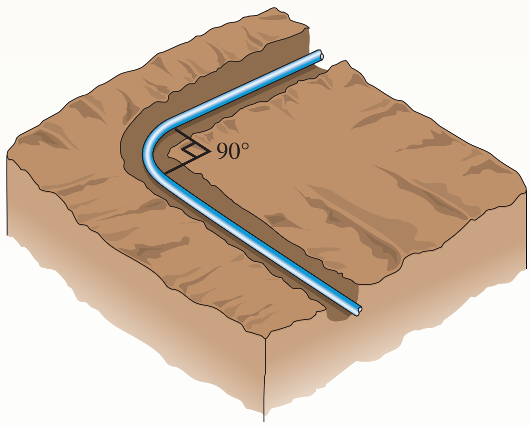
(a) If the maximum permissible strain in the copper is $\varepsilon_{max} = 0.0024$, what is the shortest length L of wire that can be used?
(b) If $L = 5.5$ m, what is the maximum acceptable diameter of the wire if the maximum normal strain must remain below yield? Assume $E = 120$ GPa and $\sigma_Y = 300$ MPa.



PROBLEM 5.4-2

5.4-3 A 4.75-in. outside diameter polyethylene pipe designed to carry chemical waste is placed in a trench and bent around a quarter-circular 90° bend (see figure). The bent section of the pipe is 52 ft long.

(a) Determine the maximum compressive strain ε_{max} in the pipe.
(b) If the normal strain cannot exceed 6.1×10^{-3}, what is the maximum diameter of the pipe?
(c) If $d = 4.75$ in., what is the minimum acceptable length of the bent section of the pipe?

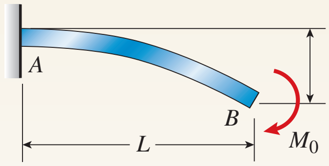


PROBLEM 5.4-3

Representative Problems

5.4-4 A cantilever beam AB is loaded by a couple M_0 at its free end (see figure). The length of the beam is $L = 2.0$ m, and the longitudinal normal strain at the top surface is $\varepsilon = 0.0010$. The distance from the top surface of the beam to the neutral surface is $c = 85$ mm.

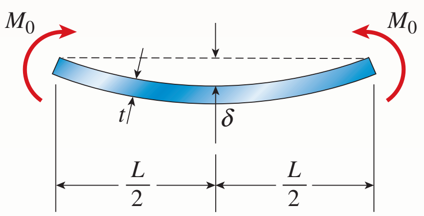
(a) Calculate the radius of curvature ρ, the curvature κ, and the vertical deflection δ at the end of the beam.
(b) If allowable strain $\varepsilon_a = 0.0008$, what is the maximum acceptable depth of the beam? [Assume that the curvature is unchanged from part(a)].
(c) If allowable strain $\varepsilon_a = 0.0008$, $c = 85$ mm, and $L = 4$ m, what is deflection δ?



PROBLEM 5.4-4

5.4-5 A thin strip of steel with a length of $L = 19$ in. and thickness of $t = 0.275$ in. is bent by couples M_0 (see figure). The deflection at the midpoint of the strip (measured from a line joining its end points) is found to be 0.30 in.

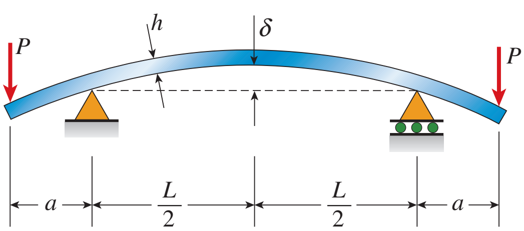
(a) Determine the longitudinal normal strain ε at the top surface of the strip.
(b) If allowable strain $\varepsilon_a = 0.0008$, what is the maximum acceptable thickness of the strip?
(c) If allowable strain $\varepsilon_a = 0.0008$, $t = 0.275$ in., and $L = 32$ in., what is deflection δ?
(d) If allowable strain $\varepsilon_a = 0.0008$, $t = 0.275$ in., and the deflection cannot exceed 1.0 in., what is the maximum permissible length of the strip?



PROBLEM 5.4-5

5.4-6 A bar of rectangular cross section is loaded and supported as shown in the figure. The distance between supports is $L = 1.75$ m, and the height of the bar is $h = 140$ mm. The deflection at the midpoint is measured as 2.5 mm.

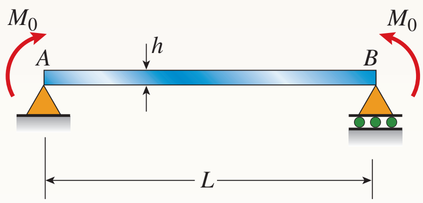
(a) What is the maximum normal strain ε at the top and bottom of the bar?



PROBLEM 5.4-6

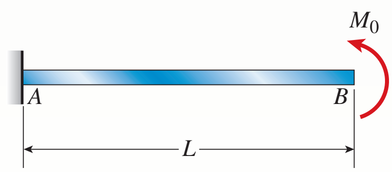
(b) If allowable strain $\varepsilon_a = 0.0006$ and the deflection cannot exceed 4.3 mm, what is the maximum permissible length of the bar?

5.4-7 A simply supported beam with a length $L = 10$ ft and height 7 in. is bent by couples M_0 into a circular arc with downward deflection δ at the midpoint. If the curvature of the beam is $0.003\ \text{ft}^{-1}$, calculate the deflection, δ, at the mid-span of the beam and the longitudinal strain at the bottom fiber given that the distance between the neutral surface and the bottom surface is 3.5 in.



PROBLEM 5.4-7

5.4-8 A cantilever beam is subjected to a concentrated moment at B. The length of the beam $L = 3$ m and the height $h = 600$ mm. The longitudinal strain at the top of the beam is 0.0005 and the distance from the neutral surface to the bottom surface of the beam is 300 mm. Find the radius of curvature, the curvature, and the deflection of the beam at B.



PROBLEM 5.4-8

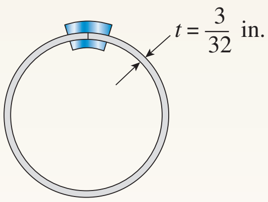
5.5 Normal Stresses in Beams (Linearly Elastic Materials)

Introductory Problems

5.5-1 A thin strip of hard copper ($E = 16{,}000$ ksi) having length $L = 90$ in. and thickness $t = 3/32$ in. is bent into a circle and held with the ends just touching (see figure).

(a) Calculate the maximum bending stress σ_{max} in the strip.
(b) By what percent does the stress increase or decrease if the thickness of the strip is increased by 1/32 in.?

(c) Find the new length of the strip so that the stress in part (b) ($t = 1/8$ in. and $L = 90$ in.) is equal to that in part (a) ($t = 3/32$ in. and $L = 90$ in.).



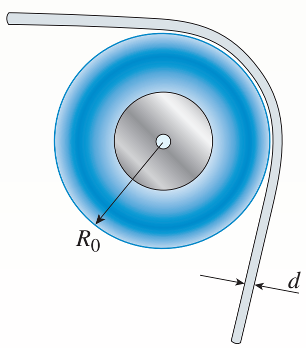
PROBLEM 5.5-1

5.5-2 A steel wire ($E = 200$ GPa) of a diameter $d = 1.25$ mm is bent around a pulley of a radius $R_0 = 500$ mm (see figure).

(a) What is the maximum stress σ_{max} in the wire?

(b) By what percent does the stress increase or decrease if the radius of the pulley is increased by 25%?

(c) By what percent does the stress increase or decrease if the diameter of the wire is increased by 25% while the pulley radius remains at $R_0 = 500$ mm?



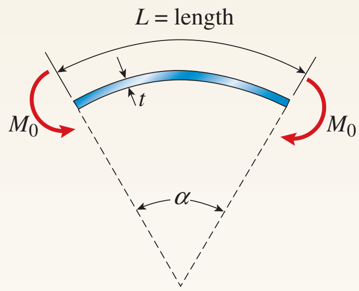
PROBLEM 5.5-2

5.5-3 A thin, high-strength steel rule ($E = 30 \times 10^6$ psi) having a thickness $t = 0.175$ in. and length $L = 48$ in. is bent by couples M_0 into a circular arc subtending a central angle $\alpha = 40°$ (see figure).

(a) What is the maximum bending stress σ_{max} in the rule?

(b) By what percent does the stress increase or decrease if the central angle is increased by 10%?

(c) What percent increase or decrease in rule thickness will result in the maximum stress reaching the allowable value of 42 ksi?



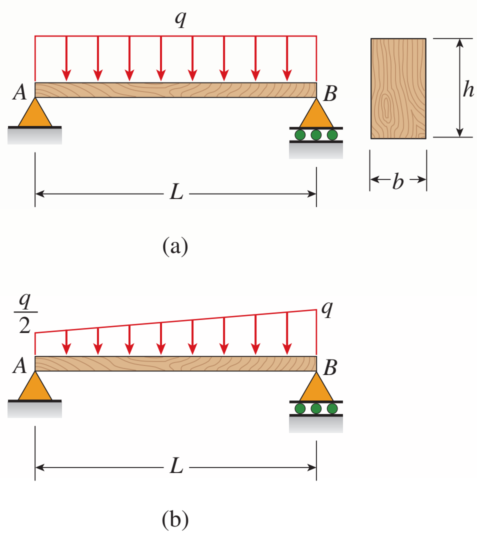
PROBLEM 5.5-3

Representative Problems

5.5-4 A simply supported wood beam AB with a span length $L = 4$ m carries a uniform load of intensity $q = 5.8$ kN/m (see figure).

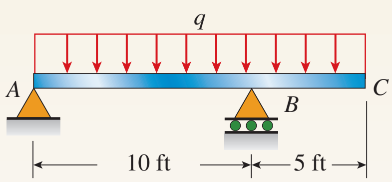
(a) Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width $b = 140$ mm and height $h = 240$ mm.

(b) Repeat part (a) but use the trapezoidal distributed load shown in the figure part b.



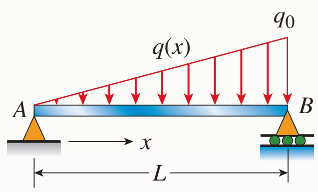
PROBLEM 5.5-4

5.5-5 Beam ABC has simple supports at A and B and an overhang from B to C. The beam is constructed from a steel W 16 × 31. The beam must carry its own weight in addition to uniform load $q = 150$ lb/ft. Determine the maximum tensile and compressive stresses in the beam.



PROBLEM 5.5-5

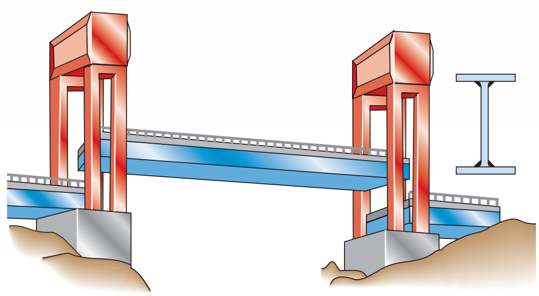
5.5-6 A simply supported beam is subjected to a linearly varying distributed load $q(x) = \frac{x}{L} q_0$ with maximum intensity q_0 at B. The beam has a length $L = 4$ m and rectangular cross section with a width of 200 mm and height of 300 mm. Determine the maximum permissible value for the maximum intensity, q_0, if the allowable normal stresses in tension and compression are 120 MPa.



PROBLEM 5.5-6

5.5-7 Each girder of the lift bridge (see figure) is 180 ft long and simply supported at the ends. The design load for each girder is a uniform load of intensity 1.6 kips/ft. The girders are fabricated by welding three steel plates to form an I-shaped cross section (see figure) having section modulus $S = 3600 \text{ in}^3$.

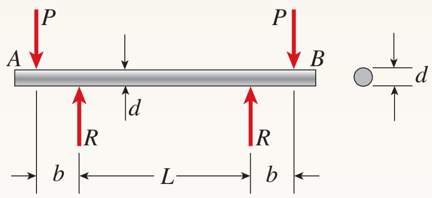
What is the maximum bending stress σ_{max} in a girder due to the uniform load?



PROBLEM 5.5-7

5.5-8 A freight-car axle AB is loaded approximately as shown in the figure, with the forces P representing the car loads (transmitted to the axle through the axle boxes) and the forces R representing the rail loads (transmitted to the axle through the wheels). The diameter of the axle is $d = 82$ mm, the distance between centers of the rails is L, and the distance between the forces P and R is $b = 220$ mm.

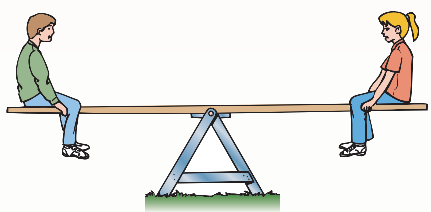
Calculate the maximum bending stress σ_{max} in the axle if $P = 50$ kN.



PROBLEM 5.5-8

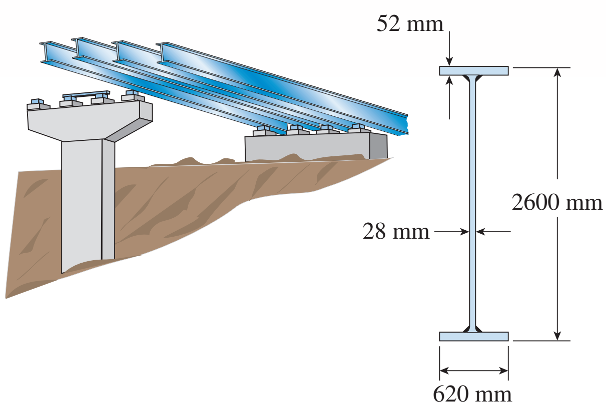
5.5-9 A seesaw weighing 3 lb/ft of length is occupied by two children, each weighing 90 lb (see figure). The center of gravity of each child is 8 ft from the fulcrum. The board is 19 ft long, 8 in. wide, and 1.5 in. thick.

What is the maximum bending stress in the board?



PROBLEM 5.5-9

5.5-10 During construction of a highway bridge, the main girders are cantilevered outward from one pier toward the next (see figure). Each girder has a cantilever length of 48 m and an I-shaped cross



PROBLEM 5.5-10

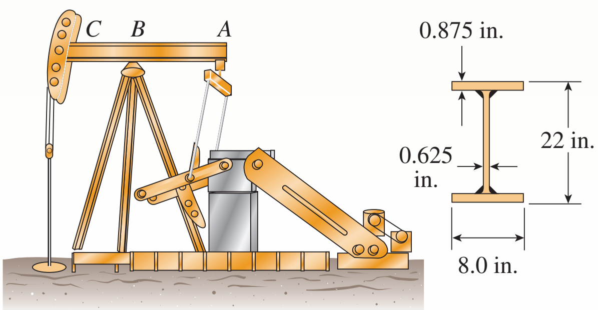
section with dimensions shown in the figure. The load on each girder (during construction) is assumed to be 9.5 kN/m, which includes the weight of the girder.

Determine the maximum bending stress in a girder due to this load.

5.5-11 The horizontal beam ABC of an oil-well pump has the cross section shown in the figure. If the vertical pumping force acting at end C is 9 kips and if the distance from the line of action of that force to point B is 16 ft, what is the maximum bending stress in the beam due to the pumping force?

Gabriel M. Covian/Getty Images

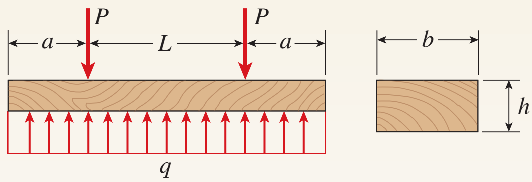
Horizontal beam transfers loads as part of oil well pump



PROBLEM 5.5-11

5.5-12 A railroad tie (or *sleeper*) is subjected to two rail loads, each of magnitude $P = 175$ kN, acting as shown in the figure. The reaction q of the ballast is assumed to be uniformly distributed over the length of the tie, which has cross-sectional dimensions $b = 300$ mm and $h = 250$ mm.

Calculate the maximum bending stress σ_{max} in the tie due to the loads P, assuming the distance $L = 1500$ mm and the overhang length $a = 500$ mm.



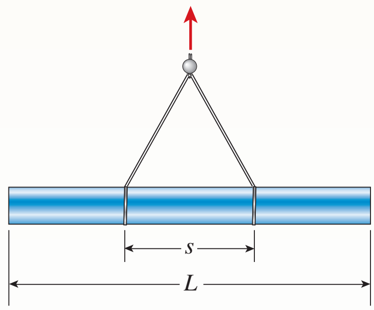
PROBLEM 5.5-12

5.5-13 A fiberglass pipe is lifted by a sling, as shown in the figure. The outer diameter of the pipe is 6.0 in., its thickness is 0.25 in., and its weight density is 0.053 lb/in^3. The length of the pipe is $L = 36$ ft and the distance between lifting points is $s = 11$ ft.

(a) Determine the maximum bending stress in the pipe due to its own weight.

(b) Find the spacing s between lift points which minimizes the bending stress. What is the minimum bending stress?

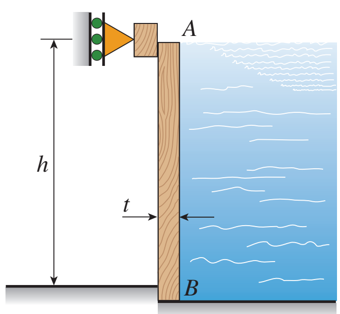
(c) What spacing s leads to maximum bending stress? What is that stress?



PROBLEM 5.5-13

5.5-14 A small dam of height $h = 2.0$ m is constructed of vertical wood beams AB of thickness $t = 120$ mm, as shown in the figure. Consider the beams to be simply supported at the top and bottom.

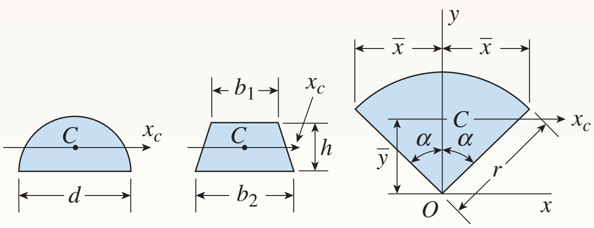
Determine the maximum bending stress σ_{max} in the beams, assuming that the weight density of water is $\gamma = 9.81$ kN/m^3.



PROBLEM 5.5-14

5.5-15 Determine the maximum tensile stress σ_t (due to pure bending about a horizontal axis through C by positive bending moments M) for beams having cross sections as follows (see figure).

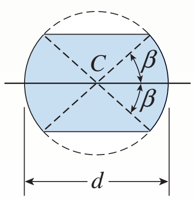
(a) A semicircle of diameter d.
(b) An isosceles trapezoid with bases $b_1 = b$ and $b_2 = 4b/3$ and altitude h.
(c) A circular sector with $\alpha = \pi/3$ and $r = d/2$



(a) (b) (c)

PROBLEM 5.5-15

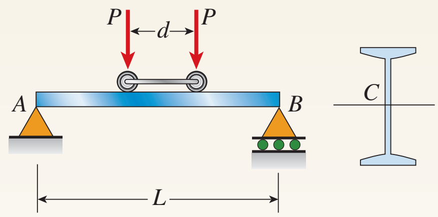
5.5-16 Determine the maximum bending stress σ_{max} (due to pure bending by a moment M) for a beam having a cross section in the form of a circular core (see figure). The circle has diameter d and the angle $\beta = 60°$. *Hint:* Use the formulas given in Appendix E, Cases 9 and 15.



PROBLEM 5.5-16

5.5-17 A simple beam AB of a span length $L = 24$ ft is subjected to two wheel loads acting at a distance $d = 5$ ft apart (see figure). Each wheel transmits a load $P = 3.0$ kips, and the carriage may occupy any position on the beam.

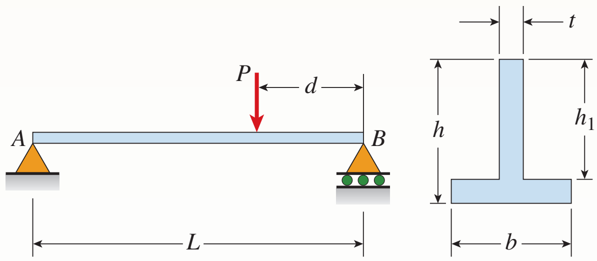
(a) Determine the maximum bending stress σ_{max} due to the wheel loads if the beam is an I-beam having section modulus $S = 16.2\ \text{in}^3$.
(b) If $d = 5$ ft, find the required span length L to reduce the maximum stress in part (a) to 18 ksi.
(c) If $L = 24$ ft, find the required wheel spacing s to reduce the maximum stress in part (a) to 18 ksi.



PROBLEM 5.5-17

5.5-18 Determine the maximum tensile stress σ_t and maximum compressive stress σ_c due to the load P acting on the simple beam AB (see figure).

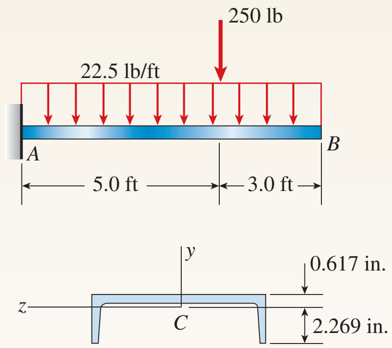
(a) Data are $P = 6.2$ kN, $L = 3.2$ m, $d = 1.25$ m, $b = 80$ mm, $t = 25$ mm, $h = 120$ mm, and $h_1 = 90$ mm.
(b) Find the value of d for which tensile and compressive stresses are the largest. What are these stresses?



PROBLEM 5.5-18

5.5-19 A cantilever beam AB, loaded by a uniform load and a concentrated load (see figure), is constructed of a channel section.

(a) Find the maximum tensile stress σ_t and maximum compressive stress σ_c if the cross section has the dimensions indicated and the moment of inertia about the z axis (the neutral axis) is $I = 3.36\ \text{in}^4$. *Note:* The uniform load represents the weight of the beam.
(b) Find the maximum value of the concentrated load if the maximum tensile stress cannot exceed 4 ksi and the maximum compressive stress is limited to 14.5 ksi.
(c) How far from A can load $P = 250$ lb be positioned if the maximum tensile stress cannot exceed 4 ksi and the maximum compressive stress is limited to 14.5 ksi?



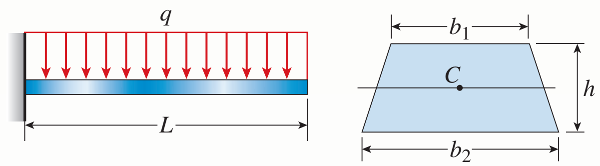
PROBLEM 5.5-19

5.5-20 A cantilever beam AB of an isosceles trapezoidal cross section has a length $L = 0.8$ m, dimensions $b_1 = 80$ mm and $b_2 = 90$ mm, and height $h = 110$ mm (see figure). The beam is made of brass weighing 85 kN/m^3.

(a) Determine the maximum tensile stress σ_t and maximum compressive stress σ_c due to the beam's own weight.

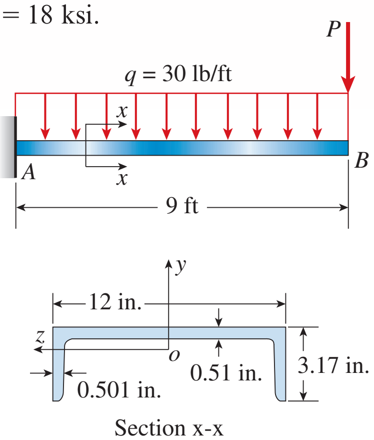
(b) If the width b_1 is doubled, what happens to the stresses?

(c) If the height h is doubled, what happens to the stresses?



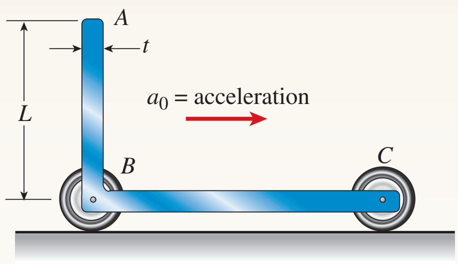
PROBLEM 5.5-20

5.5-21 A cantilever beam, a C12 × 30 section, is subjected to its own weight and a point load at B. Find the maximum permissible value of load P at B (kips) if the allowable stress in tension and compression is $\sigma_a = 18$ ksi.



PROBLEM 5.5-21

5.5-22 A frame ABC travels horizontally with an acceleration a_0 (see figure). Obtain a formula for the maximum stress σ_{max} in the vertical arm AB, which has length L, thickness t, and mass density ρ.

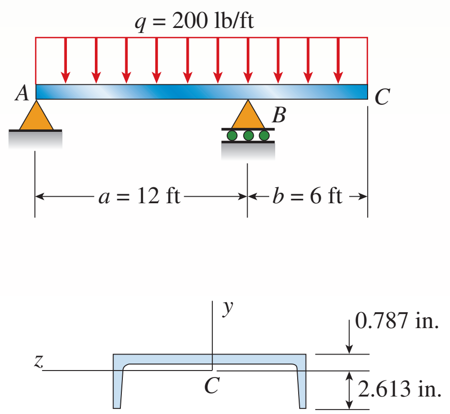


PROBLEM 5.5-22

5.5-23 A beam ABC with an overhang from B to C supports a uniform load of 200 lb/ft throughout its length (see figure). The beam is a channel section with dimensions as shown in the figure. The moment of inertia about the z axis (the neutral axis) equals 8.13 in^4.

(a) Calculate the maximum tensile stress σ_t and maximum compressive stress σ_c due to the uniform load.

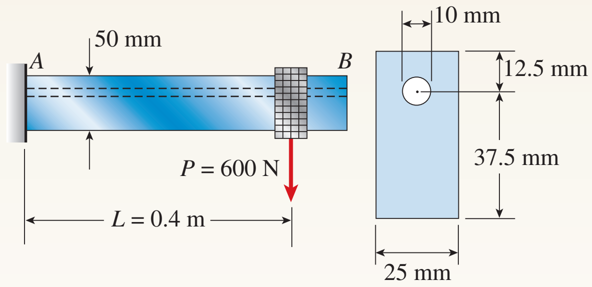
(b) Find the required span length a that results in the ratio of larger to smaller compressive stress being equal to the ratio of larger to smaller tensile stress for the beam. Assume that the total length $L = a + b = 18$ ft remains unchanged.



PROBLEM 5.5-23

5.5-24 A cantilever beam AB with a rectangular cross section has a longitudinal hole drilled throughout its length (see figure). The beam supports a load $P = 600$ N. The cross section is 25 mm wide and 50 mm high, and the hole has a diameter of 10 mm.

Find the bending stresses at the top of the beam, at the top of the hole, and at the bottom of the beam.



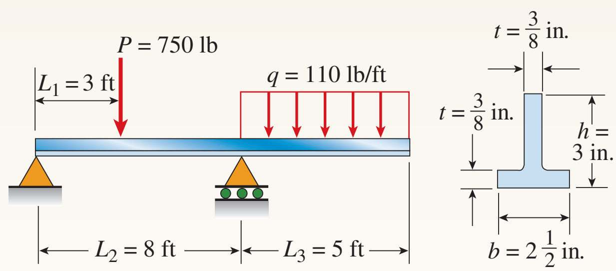
PROBLEM 5.5-24

5.5-25 A beam with a T-section is supported and loaded as shown in the figure. The cross section has width $b = 2\ 1/2$ in., height $h = 3$ in., and thickness $t = 3/8$ in.

(a) Determine the maximum tensile and compressive stresses in the beam.

(b) If the allowable stresses in tension and compression are 18 ksi and 12 ksi, respectively, what is the required depth h of the beam? Assume that thickness t remains at 3/8 in. and that flange width $b = 2.5$ in.

(c) Find the new values of loads P and q so that the allowable tension (18 ksi) and compression (12 ksi) stresses are reached simultaneously for the beam. Use the beam cross section in part (a) (see figure) and assume that L_1, L_2, and L_3 are unchanged.

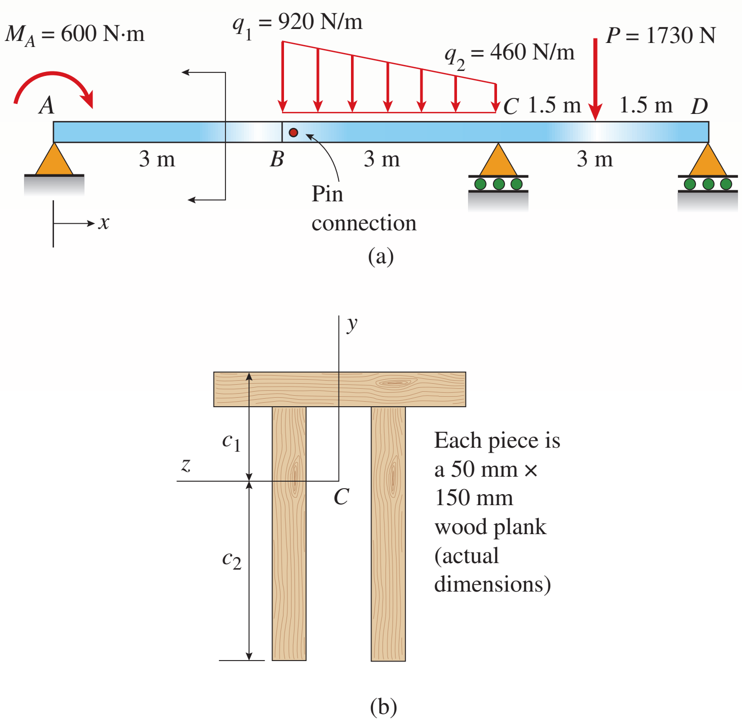


PROBLEM 5.5-25

5.5-26 Consider the compound beam with segments AB and BCD joined by a pin connection (moment release) just right of B (see figure part a). The beam cross section is a double-T made up from three 50 mm × 150 mm wood members (actual dimensions, see figure part b).

(a) Find the centroid C of the double-T cross section (c_1, c_2); then compute the moment of inertia, $[I_z(\text{mm}^4)]$.

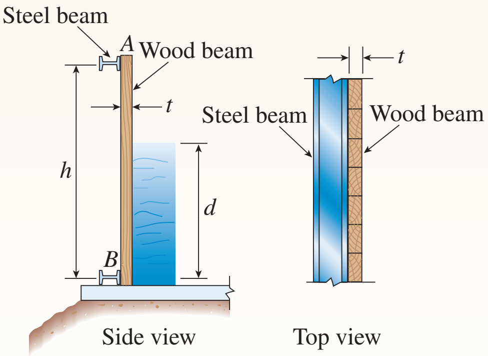
(b) Find the maximum *tensile* normal stress σ_t and maximum *compressive* normal stress σ_c (kPa) for the loading shown. (Ignore the weight of the beam.)



PROBLEM 5.5-26

5.5-27 A small dam of a height $h = 6$ ft is constructed of vertical wood beams AB, as shown in the figure. The wood beams, which have a thickness $t = 2.5$ in., are simply supported by horizontal steel beams at A and B.

Construct a graph showing the maximum bending stress σ_{max} in the wood beams versus the depth d of the water above the lower support at B. Plot the stress σ_{max} (psi) as the ordinate and the depth d (ft) as the abscissa. *Note:* The weight density γ of water equals 62.4 lb/ft^3.

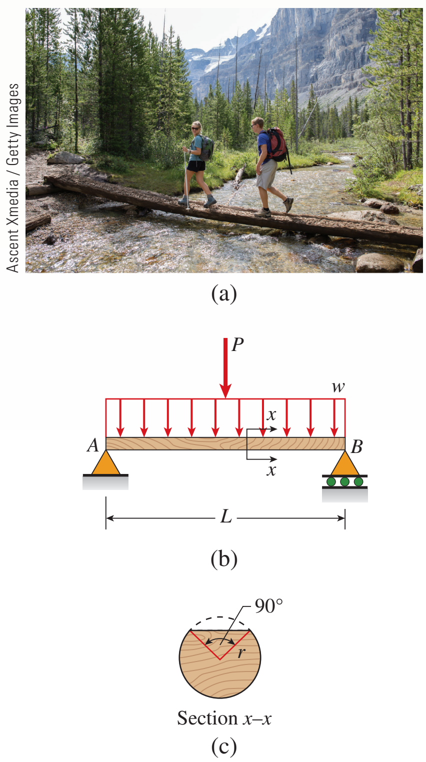


PROBLEM 5.5-27

5.5-28 A foot bridge on a hiking trail is constructed using two timber logs each having a diameter $d = 0.5$ m (see figure a). The bridge is simply supported and has a length $L = 4$ m. The top of each log is trimmed to form the walking surface (see Fig. b). A simplified model of the bridge is shown in Fig. c. *Each log* must carry its own weight $w = 1.2$ kN/m and the weight ($P = 850$ N) of a person at mid-span. (see Fig. b).

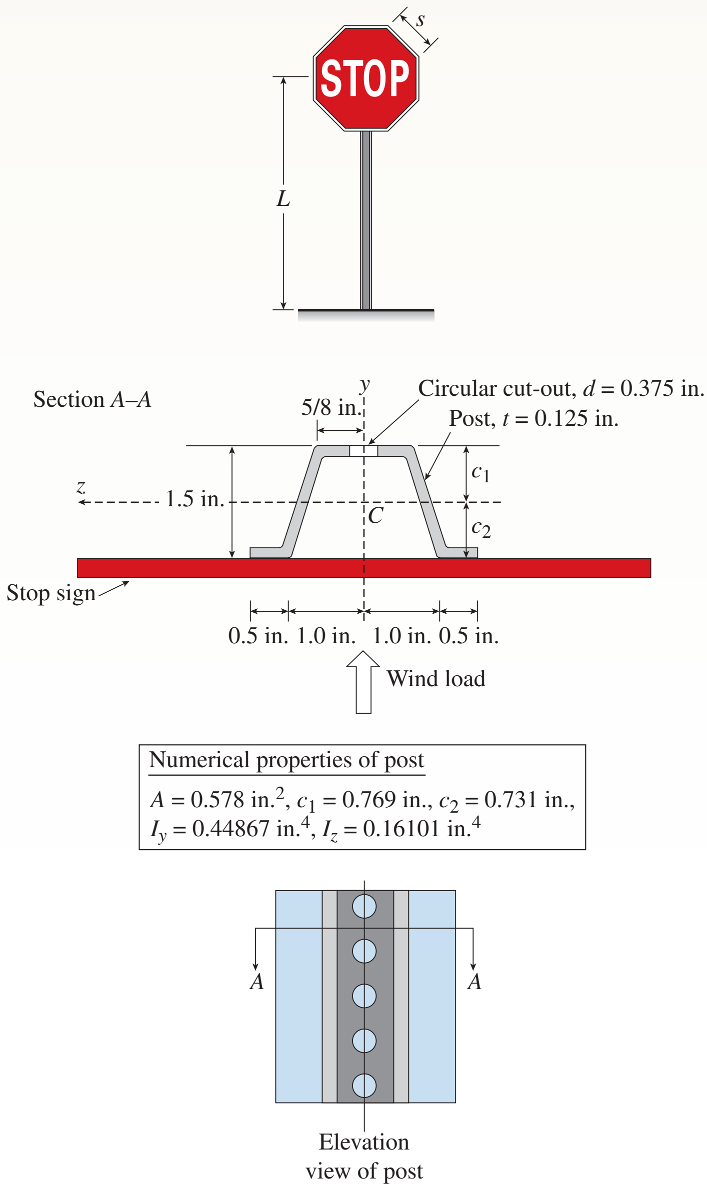
(a) Determine the maximum tensile and compressive stresses in the beam (Fig. b) due to bending.

(b) If load w is unchanged, find the maximum permissible value of load P_{max} if the allowable normal stress in tension and compression is 2.5 MPa.



PROBLEM 5.5-28

5.5-29 A steel post ($E = 30 \times 10^6$ psi) having thickness $t = 1/8$ in. and height $L = 72$ in. supports



PROBLEM 5.5-29

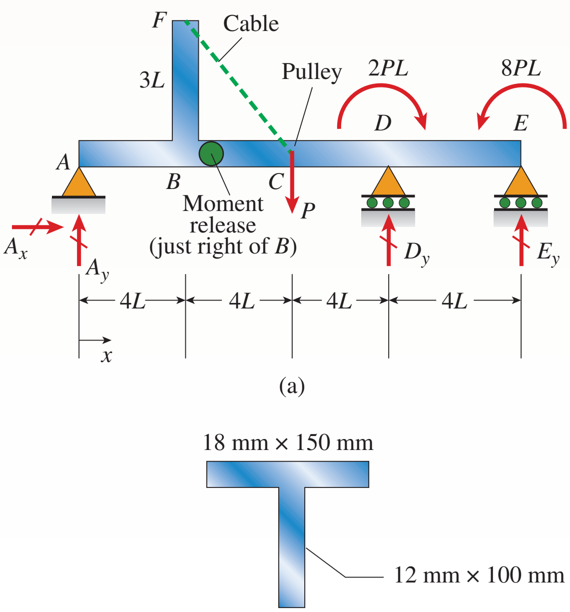
a stop sign (see figure), where $s = 12.5$ in. The height of the post L is measured from the base to the centroid of the sign. The stop sign is subjected to wind pressure $p = 20$ lb/ft^2 normal to its surface. Assume that the post is fixed at its base.

(a) What is the resultant load on the sign? (See Appendix E, Case 25, for properties of an octagon, $n = 8$.)

(b) What is the maximum bending stress σ_{max} in the post?

(c) Repeat part (b) if the circular cut-outs are eliminated over the height of the post.

5.5-30 Beam $ABCDE$ has a moment release just right of joint B and has concentrated moment loads at D and E. In addition, a cable with tension P is attached at F and runs over a pulley at C (Fig. a). The beam is constructed using two steel plates, which are welded to form a T cross section (see Fig. b). *Consider flexural stresses only.* Find the maximum permissible value of load *variable* P if the allowable bending stress is 130 MPa. Ignore the self-weight of the frame members and let length *variable* $L = 0.75$ m.



(a)

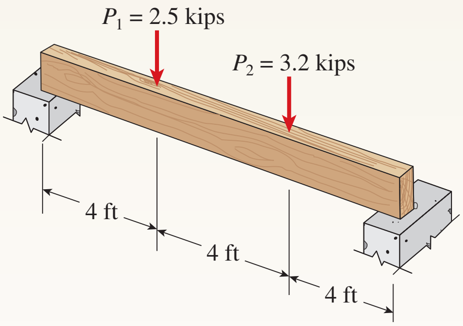
(b)

PROBLEM 5.5-30

5.6 Design of Beams for Bending Stresses

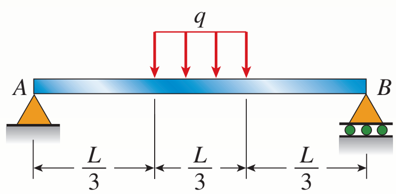
Introductory Problems

5.6-1 A simply supported wood beam having a span length $L = 12$ ft is subjected to unsymmetrical point loads, as shown in the figure. Select a suitable size for the beam from the table in Appendix G. The allowable bending stress is 1800 psi and the wood weighs 35 lb/ft^3.



PROBLEM 5.6-1

5.6-2 A simply supported beam ($L = 4.5$ m) must support mechanical equipment represented as a distributed load with intensity $q = 30$ kN/m acting over the middle segment of the beam (see figure). Select the most economical W-shape steel beam from Table F-1(b) to support the loads. Consider both the distributed force q *and the weight of the beam*. Use an allowable bending stress of 140 MPa.

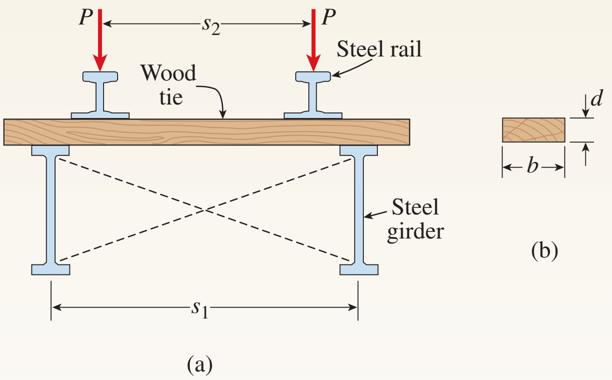


PROBLEM 5.6-2

5.6-3 The cross section of a narrow-gage railway bridge is shown in part a of the figure. The bridge is constructed with longitudinal steel girders that support the wood cross ties. The girders are restrained against lateral buckling by diagonal bracing, as indicated by the dashed lines.

The spacing of the girders is $s_1 = 50$ in. and the spacing of the rails is $s_2 = 30$ in. The load transmitted by each rail to a single tie is $P = 1500$ lb. The cross section of a tie, shown in part b of the figure, has a width $b = 5.0$ in. and depth d.

Determine the minimum value of d based upon an allowable bending stress of 1125 psi in the wood tie. (Disregard the weight of the tie itself.)

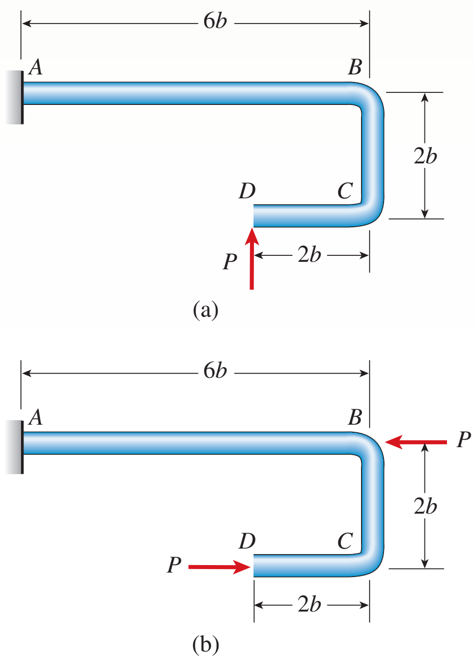


PROBLEM 5.6-3

5.6-4 A fiberglass bracket *ABCD* with a solid circular cross section has the shape and dimensions shown in the figure. A vertical load $P = 40$ N acts at the free end *D*.

(a) Determine the minimum permissible diameter d_{min} of the bracket if the allowable bending stress in the material is 30 MPa and $b = 37$ mm. *Note:* Disregard the weight of the bracket itself.

(b) If $d = 10$ mm, $b = 37$ mm, and $\sigma_{allow} = 30$ MPa, what is the maximum value of load *P* if vertical load *P* at *D* is replaced with horizontal loads *P* at *B* and *D* (see figure part b)?



PROBLEM 5.6-4

Representative Problems

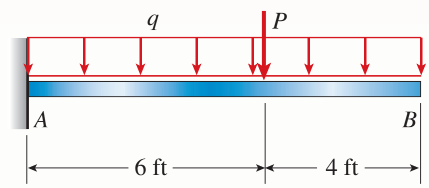
5.6-5 A cantilever beam *AB* is loaded by a uniform load *q* and a concentrated load *P*, as shown in the figure.

(a) Select the most economical steel C shape from Table F-3(a) in Appendix F; use $q = 20$ lb/ft and $P = 300$ lb (assume allowable normal stress is $\sigma_a = 18$ ksi).

(b) Select the most economical steel S shape from Table F-2(a) in Appendix F; use $q = 45$ lb/ft and $P = 2000$ lb (assume allowable normal stress is $\sigma_a = 20$ ksi).

(c) Select the most economical steel W shape from Table F-1(a) in Appendix F; use $q = 45$ lb/ft and $P = 2000$ lb (assume allowable normal stress is $\sigma_a = 20$ ksi). *However*, assume that the design requires that the W shape must be used in weak axis bending, i.e., it must bend about the 2–2 (or *y*) axis of the cross section.

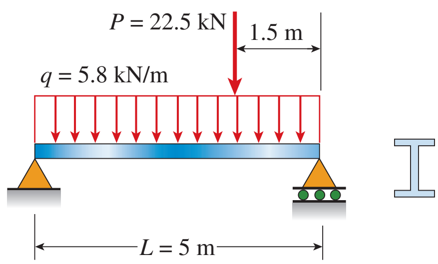
Note: For parts (a), (b), and (c), revise your initial beam selection as needed to include the distributed weight of the beam in addition to uniform load *q*.



PROBLEM 5.6-5

5.6-6 A simple beam of length $L = 5$ m carries a uniform load of intensity $q = 5.8$ kN/m and a concentrated load 22.5 kN (see figure).

(a) Assuming $\sigma_{allow} = 110$ MPa, calculate the required section modulus *S*. Then select the most economical wide-flange beam (W shape) from Table F-1(b) in Appendix F, and



PROBLEM 5.6-6

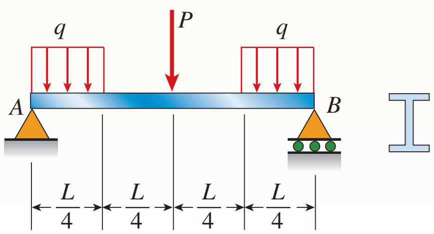
recalculate S, taking into account the weight of the beam. Select a new beam if necessary.

(b) Repeat part (a), but now assume that the design requires that the W shape must be used in weak axis bending (i.e., it must bend about the 2–2 (or y) axis of the cross section).

5.6-7 A simple beam AB is loaded as shown in the figure.

(a) Calculate the required section modulus S if $\sigma_{allow} = 18{,}000$ psi, $L = 32$ ft, $P = 2900$ lb, and $q = 450$ lb/ft. Then select a suitable I-beam (S shape) from Table F-2(a), Appendix F, and recalculate S taking into account the weight of the beam. Select a new beam size if necessary.

(b) What is the maximum load P that can be applied to your final beam selection in part (a)?

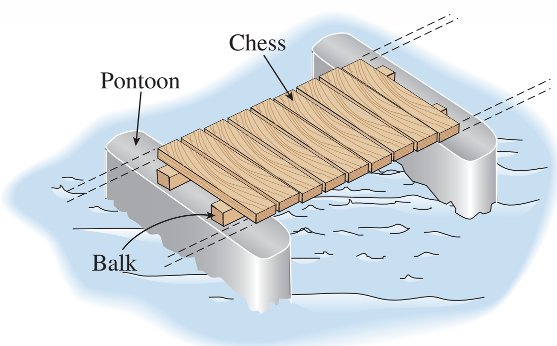


PROBLEM 5.6-7

5.6-8 A pontoon bridge (see figure) is constructed of two longitudinal wood beams, known as *balks*, that span between adjacent pontoons and support the transverse floor beams, which are called *chesses*. For purposes of design, assume that a uniform floor load of 7.5 kPa acts over the chesses. (This load includes an allowance for the weights of the chesses and balks.) Also, assume that the chesses are 2.5 m long and that the balks are simply supported with a span of 3.0 m. The allowable bending stress in the wood is 15 MPa.

(a) If the balks have a square cross section, what is their minimum required width b_{min}?

(b) Repeat part (a) if the balk width is 1.5 b and the balk depth is b; compare the cross-sectional areas of the two designs.

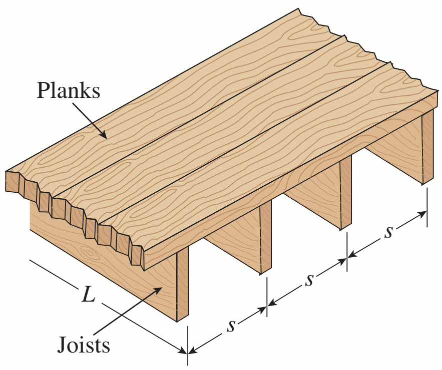


PROBLEM 5.6-8

5.6-9 A floor system in a small building consists of wood planks supported by 2-in. (nominal width) joists spaced at distance s and measured from center to center (see figure). The span length L of each joist is 12 ft, the spacing s of the joists is 16 in., and the allowable bending stress in the wood is 1250 psi. The uniform floor load is 120 lb/ft^2, which includes an allowance for the weight of the floor system itself.

(a) Calculate the required section modulus S for the joists, and then select a suitable joist size (surfaced lumber) from Appendix G, assuming that each joist may be represented as a simple beam carrying a uniform load.

(b) What is the maximum floor load that can be applied to your final beam selection in part (a)?



PROBLEMS 5.6-9 and 5.6-10

5.6-10 The wood joists supporting a plank floor (see figure) are 38 mm × 220 mm in cross section (actual dimensions) and have a span length of $L = 4.0$ m. The floor load is 5.0 kPa, which includes the weight of the joists and the floor.

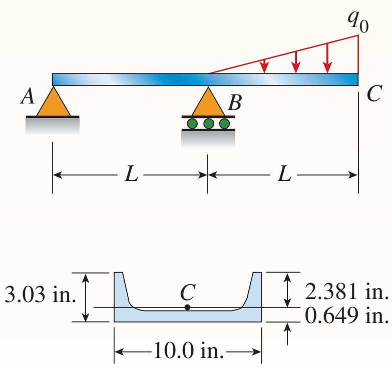
(a) Calculate the maximum permissible spacing s of the joists if the allowable bending stress is 14 MPa. (Assume that each joist may be represented as a simple beam carrying a uniform load.)

(b) If spacing $s = 406$ mm, what is the required depth h of the joist? Assume all other variables remain unchanged.

5.6-11 A beam ABC with an overhang from B to C is constructed of a C 10 × 30 channel section with flanges facing upward (see figure). The beam supports its own weight (30 lb/ft) plus a *triangular* load of maximum intensity q_0 acting on the overhang. The allowable stresses in tension and compression are 18 ksi and 12 ksi, respectively.

(a) Determine the allowable *triangular* load intensity $q_{0,\text{allow}}$ if the distance L equals 4 ft.

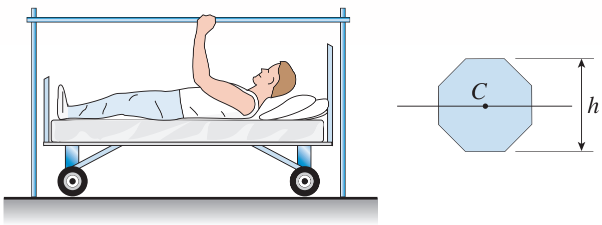
(b) What is the allowable triangular load intensity $q_{0,\text{allow}}$ if the beam is rotated 180° about its longitudinal centroidal axis so that the flanges are downward?



PROBLEM 5.6-11

5.6-12 A "trapeze bar" in a hospital room provides a means for patients to exercise while in bed (see figure). The bar is 2.1 m long and has a cross section in the shape of a regular octagon. The design load is 1.2 kN applied at the midpoint of the bar, and the allowable bending stress is 200 MPa.

Determine the minimum height h of the bar. (Assume that the ends of the bar are simply supported and that the weight of the bar is negligible.)

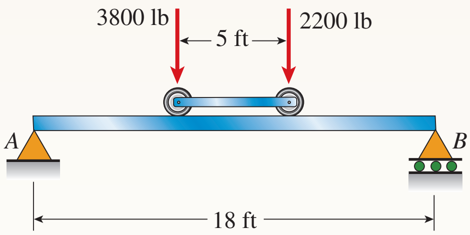


PROBLEM 5.6-12

5.6-13 A two-axle carriage that is part of an overhead traveling crane in a testing laboratory moves slowly across a simple beam AB (see figure). The load transmitted to the beam from the front axle is 2200 lb and from the rear axle is 3800 lb. The weight of the beam itself may be disregarded.

(a) Determine the minimum required section modulus S for the beam if the allowable bending stress is 17.0 ksi, the length of the beam is 18 ft, and the wheelbase of the carriage is 5 ft.

(b) Select the most economical I-beam (S shape) from Table F-2(a), Appendix F.

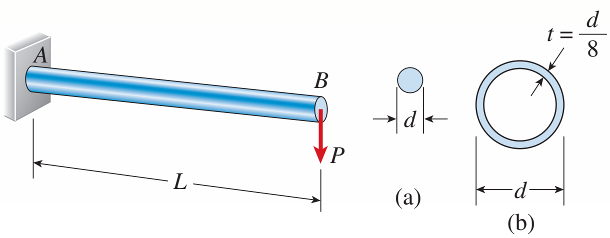


PROBLEM 5.6-13

5.6-14 A cantilever beam AB with a circular cross section and length $L = 750$ mm supports a load $P = 800$ N acting at the free end (see figure). The beam is made of steel with an allowable bending stress of 120 MPa.

(a) Determine the required diameter d_{min} (figure part a) of the beam, considering the effect of the beam's own weight.

(b) Repeat part (a) if the beam is hollow with wall thickness $t = d/8$ (figure part b); compare the cross-sectional areas of the two designs.

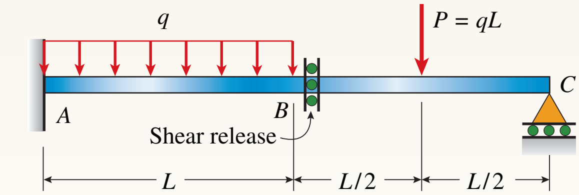


PROBLEM 5.6-14

5.6-15 A propped cantilever beam ABC (see figure) has a shear release just right of the mid-span.

(a) Select the most economical wood beam from the table in Appendix G; assume $q = 55$ lb/ft, $L = 16$ ft, $\sigma_{aw} = 1750$ psi, and $\tau_{aw} = 375$ psi. Include the self-weight of the beam in your design.

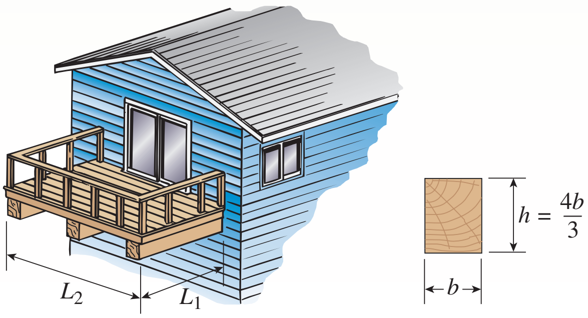
(b) If a C 10 × 25 steel beam is now used for beam ABC, what is the maximum permissible value of load variable q? Assume $\sigma_{as} = 16$ ksi and $L = 10$ ft. Include the self-weight of the beam in your analysis.



PROBLEM 5.6-15

5.6-16 A small balcony constructed of wood is supported by three identical cantilever beams (see figure). Each beam has length $L_1 = 2.1$ m, width b, and height $h = 4b/3$. The dimensions of the balcony floor are $L_1 \times L_2$, where $L_2 = 2.5$ m. The design load is 5.5 kPa acting over the entire floor area. (This load accounts for all loads except the weights of the cantilever beams, which have a weight density $\gamma = 5.5$ kN/m^3.) The allowable bending stress in the cantilevers is 15 MPa.

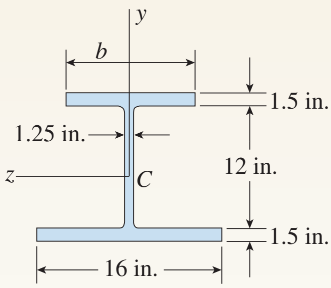
Assuming that the middle cantilever supports 50% of the load and each outer cantilever supports 25% of the load, determine the required dimensions b and h.



PROBLEM 5.6-16

5.6-17 A beam having a cross section in the form of an unsymmetric wide-flange shape (see figure) is subjected to a negative bending moment acting about the z axis.

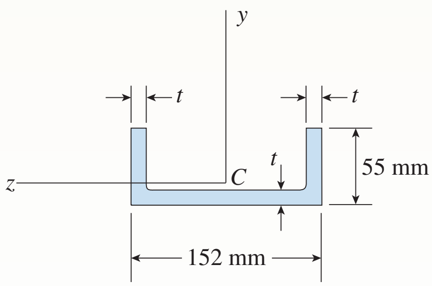
Determine the width b of the top flange in order that the stresses at the top and bottom of the beam will be in the ratio 4:3, respectively.



PROBLEM 5.6-17

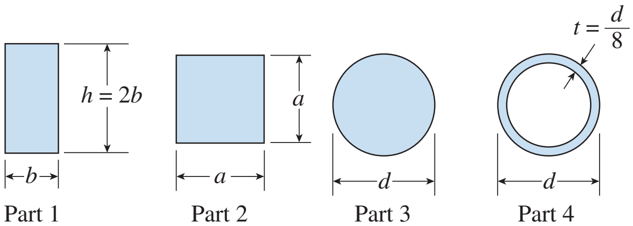
5.6-18 A beam having a cross section in the form of a channel (see figure) is subjected to a bending moment acting about the z axis.

Calculate the thickness t of the channel in order that the bending stresses at the top and bottom of the beam will be in the ratio 7:3, respectively.



PROBLEM 5.6-18

5.6-19 Determine the ratios of the weights of four beams that have the same length, are made of the same material, are subjected to the same maximum bending moment, and have the same maximum bending stress if their cross sections are (1) a rectangle with height equal to twice the width, (2) a square, (3) a circle, and (4) a pipe with outer diameter d and wall thickness $t = d/8$ (see figures).

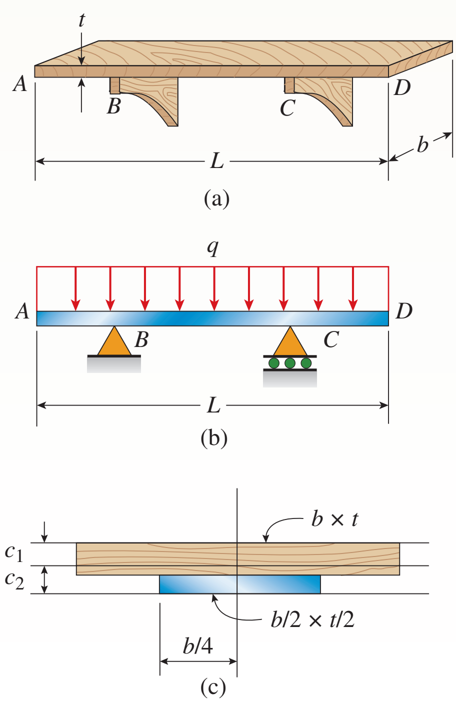


PROBLEM 5.6-19

5.6-20 A horizontal shelf AD of length $L = 1215$ mm, width $b = 305$ mm, and thickness $t = 22$ mm is supported by brackets at B and C (see part a of the figure). The brackets are adjustable and may be placed in any desired positions between the ends of the shelf. A uniform load of intensity q, which includes the weight of the shelf itself, acts on the shelf (see part b of the figure).

(a) Determine the maximum permissible value of the load q if the allowable bending stress in the shelf is $\sigma_{allow} = 8.5$ MPa and the position of the supports is adjusted for maximum load carrying capacity.

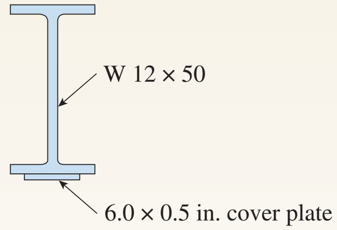
(b) The bookshelf owner decides to reinforce the shelf with a bottom wood plate $b/2 \times t/2$ along its entire length (see figure part c). Find the new maximum permissible value of the load q if the allowable bending stress in the shelf remains at $\sigma_{allow} = 8.5$ MPa.



PROBLEM 5.6-20

5.6-21 A steel plate (called a *cover plate*) having cross-sectional dimensions 6.0 in. × 0.5 in. is welded along the full length of the bottom flange of a W 12 × 50 wide-flange beam (see figure, which shows the beam cross section).

What is the percent increase in the smaller section modulus (as compared to the wide-flange beam alone)?

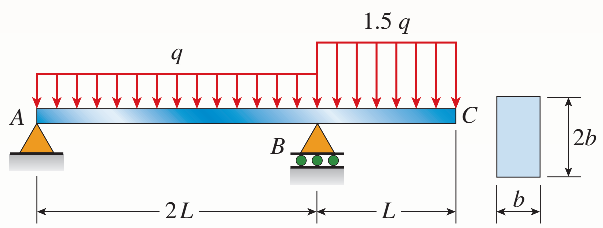


PROBLEM 5.6-21

5.6-22 A steel beam ABC is simply supported at A and B and has an overhang BC of length $L = 150$ mm (see figure). The beam supports a uniform load of intensity $q = 4.0$ kN/m over its entire span AB and $1.5q$ over BC. The cross section of the beam is rectangular with width b and height $2b$. The allowable bending stress in the steel is $\sigma_{allow} = 60$ MPa, and its weight density is $\gamma = 77.0$ kN/m^3.

(a) Disregarding the weight of the beam, calculate the required width b of the rectangular cross section.

(b) Taking into account the weight of the beam, calculate the required width b.



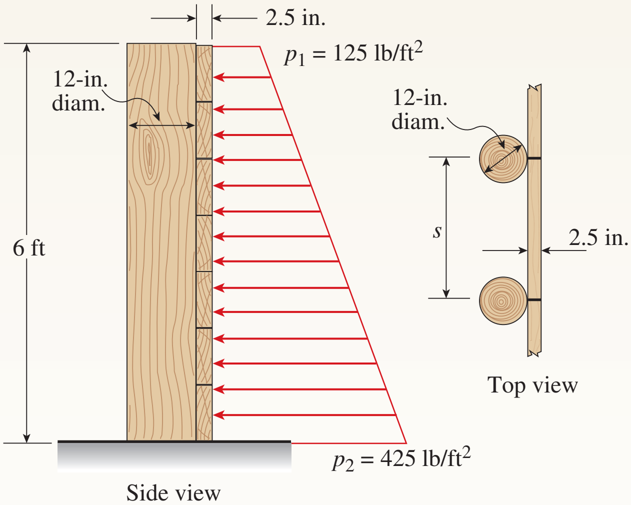
PROBLEM 5.6-22

5.6-23 A retaining wall 6 ft high is constructed of horizontal wood planks 2.5 in. thick (actual dimension) that are supported by vertical wood piles of a 12 in. diameter (actual dimension), as shown in the figure. The lateral earth pressure is $p_1 = 125$ lb/ft^2 at the top of the wall and $p_2 = 425$ lb/ft^2 at the bottom.

(a) Assuming that the allowable stress in the wood is 1175 psi, calculate the maximum permissible spacing s of the piles.

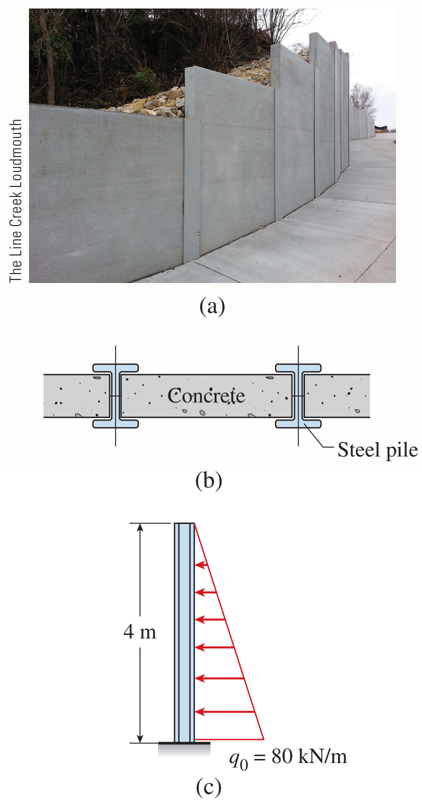
(b) Find the required diameter of the wood piles so that piles and planks ($t = 2.5$ in.) reach the allowable stress at the same time.

Hint: Observe that the spacing of the piles may be governed by the load-carrying capacity of either the planks or the piles. Consider the piles to act as cantilever beams subjected to a trapezoidal distribution of load, and consider the planks to act as simple beams between the piles. To be on the safe side, assume that the pressure on the bottom plank is uniform and equal to the maximum pressure.



PROBLEM 5.6-23

5.6-24 A retaining wall (Fig. a) is constructed using steel W-shape columns and concrete panel infill (Fig. b). Each column is subjected to lateral soil pressure with peak intensity q_0 (Figs. b and c). The tensile and compressive strength of the beam is 600 MPa. Select the most economical W 360 section from Table F-1(b) based on safety factor of 3.0.

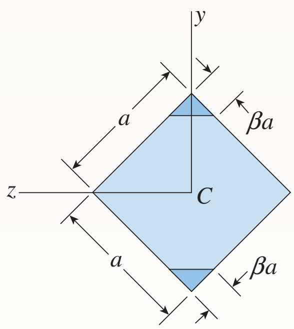


PROBLEM 5.6-24

5.6-25 A beam of square cross section (a = length of each side) is bent in the plane of a diagonal (see figure). By removing a small amount of material at the top and bottom corners, as shown by the shaded triangles in the figure, you can increase the section modulus and obtain a stronger beam, even though the area of the cross section is reduced.

(a) Determine the ratio β defining the areas that should be removed in order to obtain the strongest cross section in bending.

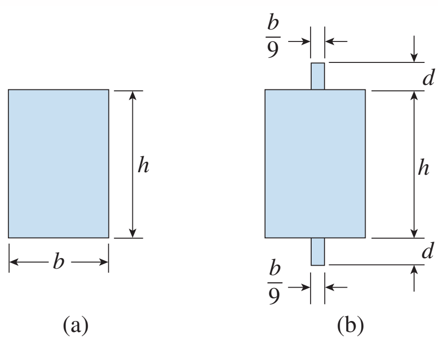
(b) By what percent is the section modulus increased when the areas are removed?



PROBLEM 5.6-25

5.6-26 The cross section of a rectangular beam having a width b and height h is shown in part a of the figure. For reasons unknown to the beam designer, it is planned to add structural projections of width $b/9$ and height d to the top and bottom of the beam (see part b of the figure).

For what values of d is the bending-moment capacity of the beam increased? For what values is it decreased?



PROBLEM 5.6-26

5.7 Nonprismatic Beams

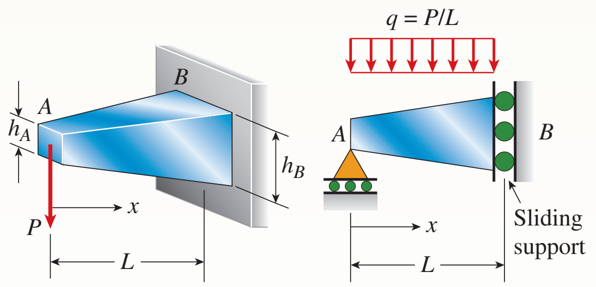
Introductory Problems

5.7-1 A tapered cantilever beam AB of length L has square cross sections and supports a concentrated load P at the free end (see figure part a). The width and height of the beam vary linearly from h_A at the free end to h_B at the fixed end.

Determine the distance x from the free end A to the cross section of maximum bending stress if $h_B = 3h_A$.

(a) What is the magnitude σ_{max} of the maximum bending stress? What is the ratio of the maximum stress to the largest stress B at the support?

(b) Repeat part (a) if load P is now applied as a uniform load of intensity $q = P/L$ over the entire beam, A is restrained by a roller support, and B is a sliding support (see figure part b).



(a) (b)

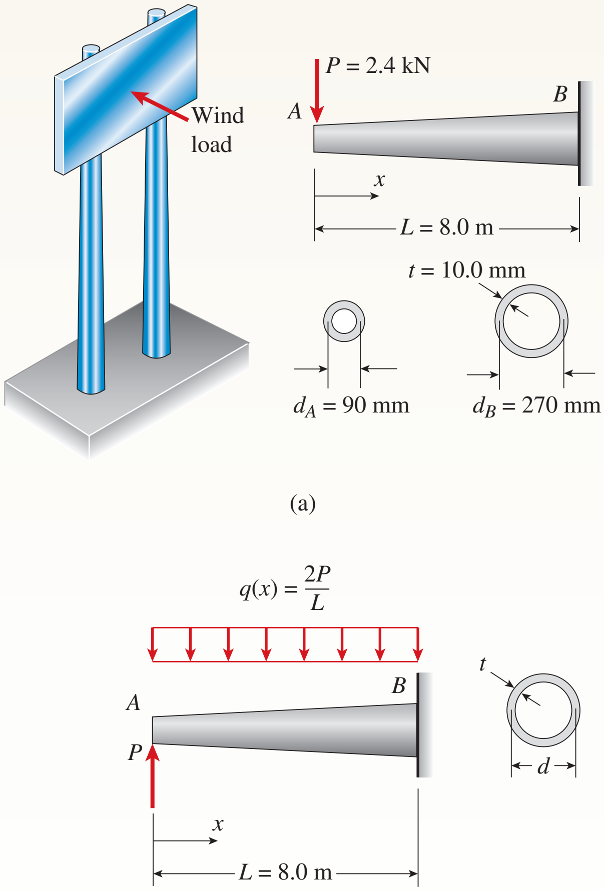
PROBLEM 5.7-1

5.7-2 A tall signboard is supported by two vertical beams consisting of thin-walled, tapered circular tubes (see figure part a). For purposes of this analysis, each beam may be represented as a cantilever AB of length $L = 8.0$ m subjected to a lateral load $P = 2.4$ kN at the free end. The tubes have a constant thickness $t = 10.0$ mm and average diameters $d_A = 90$ mm and $d_B = 270$ mm at ends A and B, respectively.

Because the thickness is small compared to the diameters, the moment of inertia at any cross section may be obtained from the formula $I = \pi d^3 t/8$ (see Case 22, Appendix E); therefore, the section modulus may be obtained from the formula $S = \pi d^2 t/4$.

(a) At what distance x from the free end does the maximum bending stress occur? What is the magnitude σ_{max} of the maximum bending stress? What is the ratio of the maximum stress to the largest stress σ_B at the support?

(b) Repeat part (a) if concentrated load P is applied upward at A and downward uniform load $q(x) = 2P/L$ is applied over the entire beam as shown in the figure part b. What is the ratio of the maximum stress to the stress at the location of maximum moment?



(b)

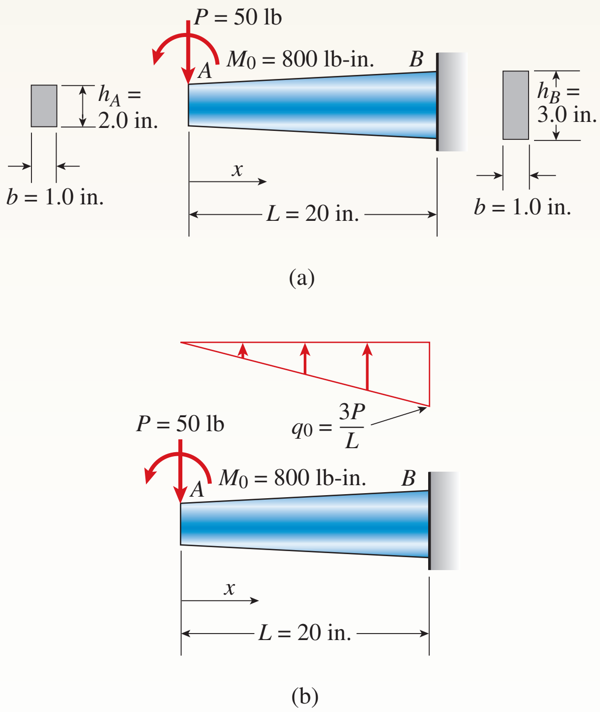
PROBLEM 5.7-2

Representative Problems

5.7-3 A tapered cantilever beam AB with rectangular cross sections is subjected to a concentrated load $P = 50$ lb and a couple $M_0 = 800$ lb-in. acting at the free end (see figure part a). The width b of the beam is constant and equal to 1.0 in., but the height varies linearly from $h_A = 2.0$ in. at the loaded end to $h_B = 3.0$ in. at the support.

(a) At what distance x from the free end does the maximum bending stress σ_{max} occur? What is the magnitude σ_{max} of the maximum bending stress? What is the ratio of the maximum stress to the largest stress σ_B at the support?

(b) Repeat part a if, in addition to P and M_0, a triangular distributed load with peak intensity $q_0 = 3P/L$ acts upward over the entire beam as shown in the figure part b. What is the ratio of the maximum stress to the stress at the location of maximum moment?

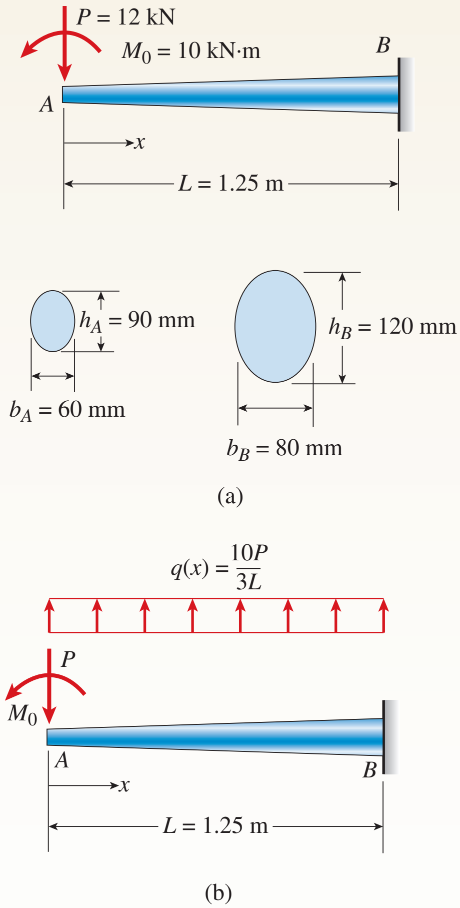


PROBLEM 5.7-3

5.7-4 The spokes in a large flywheel are modeled as beams fixed at one end and loaded by a force P and a couple M_0 at the other (see figure). The cross sections of the spokes are elliptical with major and minor axes (height and width, respectively) having the lengths shown in the figure part a. The cross-sectional dimensions vary linearly from end A to end B.

Considering only the effects of bending due to the loads P and M_0, determine the following quantities.

(a) The largest bending stress σ_A at end A.
(b) The largest bending stress σ_B at end B.
(c) The distance x to the cross section of maximum bending stress.
(d) The magnitude σ_{max} of the maximum bending stress.
(e) Repeat part d if a uniform load $q(x) = 10P/3L$ is added to loadings P and M_0, as shown in the figure part b.



PROBLEM 5.7-4

5.7-5 Refer to the tapered cantilever beam of solid circular cross section shown in Fig. 5-26 of Example 5-9.

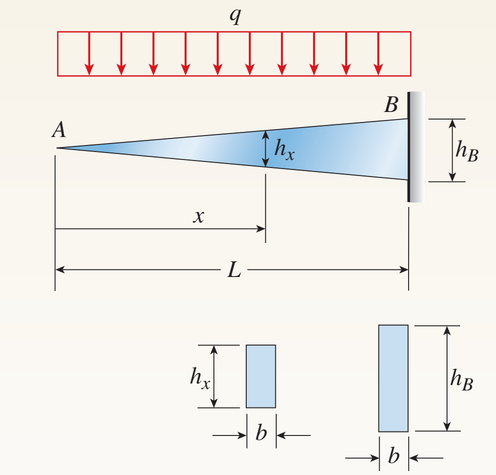
(a) Considering only the bending stresses due to the load P, determine the range of values of the ratio d_B/d_A for which the maximum normal stress occurs at the support.
(b) What is the maximum stress for this range of values?

Fully Stressed Beams

Problems 5.7-6 to 5.7-8 pertain to fully stressed beams of rectangular cross section. Consider only the bending stresses obtained from the flexure formula and disregard the weights of the beams.

5.7-6 A cantilever beam AB with rectangular cross sections of a constant width b and varying height h_x is subjected to a uniform load of intensity q (see figure).

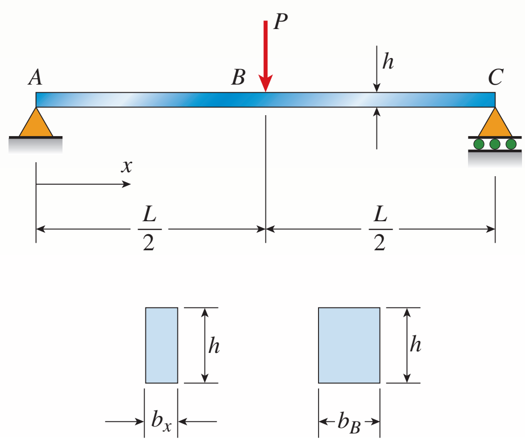
How should the height h_x vary as a function of x (measured from the free end of the beam) in order to have a fully stressed beam? (Express h_x in terms of the height h_B at the fixed end of the beam.)



PROBLEM 5.7-6

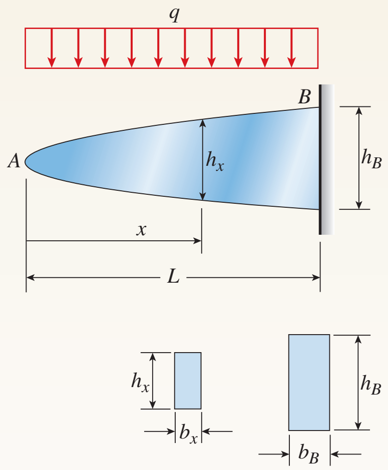
5.7-7 A simple beam ABC having rectangular cross sections with constant height h and varying width b_x supports a concentrated load P acting at the midpoint (see figure).

How should the width b_x vary as a function of x in order to have a fully stressed beam? (Express b_x in terms of the width b_B at the midpoint of the beam.)



PROBLEM 5.7-7

5.7-8 A cantilever beam AB having rectangular cross sections with varying width b_x and varying height h_x is subjected to a uniform load of intensity q (see figure). If the width varies linearly with x according to the equation $b_x = b_B x/L$, how should the height h_x vary as a function of x in order to have a fully stressed beam? (Express h_x in terms of the height h_B at the fixed end of the beam.)



PROBLEM 5.7-8

5.8 Shear Stresses in Beams of Rectangular Cross Section

Introductory Problems

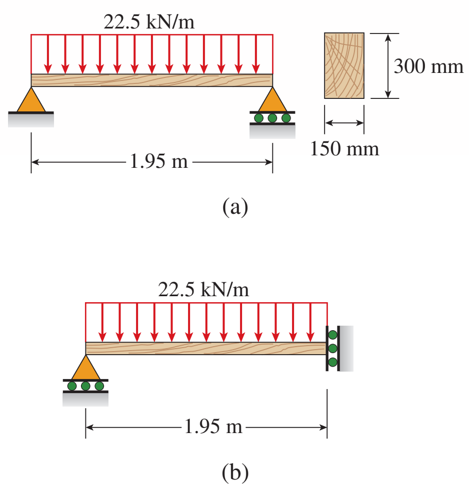
5.8-1 The shear stresses τ in a rectangular beam are given by Eq. (5-43):

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

in which V is the shear force, I is the moment of inertia of the cross-sectional area, h is the height of the beam, and y_1 is the distance from the neutral axis to the point where the shear stress is being determined (Fig. 5-32).

By integrating over the cross-sectional area, show that the resultant of the shear stresses is equal to the shear force V.

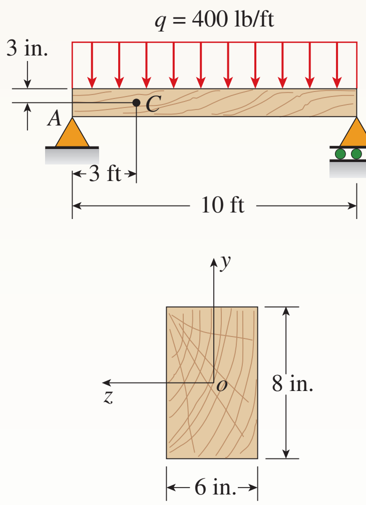
5.8-2 Calculate the maximum shear stress τ_{max} and the maximum bending stress σ_{max} in a wood beam



PROBLEM 5.8-2

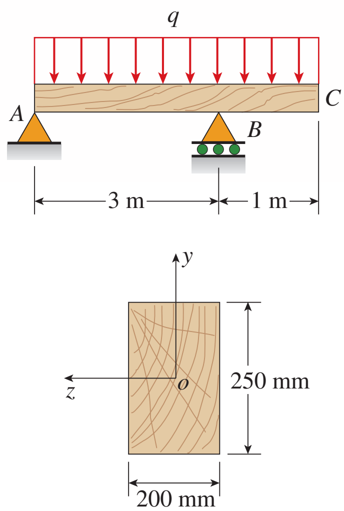
(see figure) carrying a uniform load of 22.5 kN/m (which includes the weight of the beam) if the length is 1.95 m and the cross section is rectangular with width 150 mm and height 300 mm, and the beam is either (a) simply supported as in the figure part a, or b has a sliding support at right as in the figure part b.

5.8-3 A simply supported wood beam is subjected to uniformly distributed load q. The width of the beam is 6 in. and the height is 8 in. Determine the normal stress and the shear stress at point C. Show these stresses on a sketch of a stress element at point C.



PROBLEM 5.8-3

5.8-4 A simply supported wood beam with overhang is subjected to uniformly distributed load q. The beam has a rectangular cross section with width $b = 200$ mm and height $h = 250$ mm. Determine the maximum permissible value q if the allowable bending stress is $\sigma_{all} = 11$ MPa, and the allowable shear stress is $\tau_{all} = 1.2$ MPa.

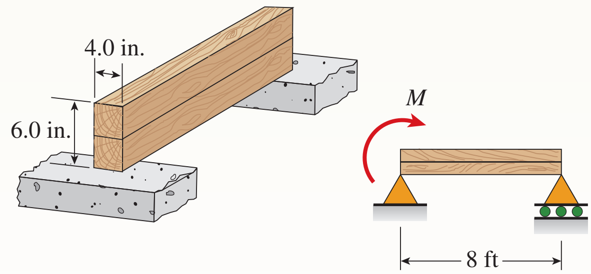


PROBLEM 5.8-4

5.8-5 Two wood beams, each of rectangular cross section (3.0 in. × 4.0 in., actual dimensions), are glued together to form a solid beam with dimensions 6.0 in. × 4.0 in. (see figure). The beam is simply supported with a span of 8 ft.

(a) What is the maximum moment M_{max} that may be applied at the left support if the allowable shear stress in the glued joint is 200 psi? (Include the effects of the beam's own weight, assuming that the wood weighs 35 lb/ft^3.)

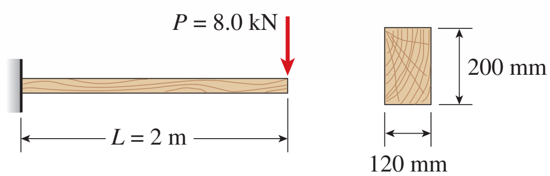
(b) Repeat part (a) if M_{max} is based on allowable bending stress of 2500 psi.



PROBLEM 5.8-5

5.8-6 A cantilever beam of length $L = 2$ m supports a load $P = 8.0$ kN (see figure). The beam is made of wood with cross-sectional dimensions 120 mm × 200 mm.

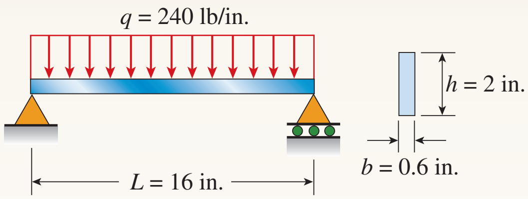
Calculate the shear stresses due to the load P at points located 25 mm, 50 mm, 75 mm, and 100 mm from the top surface of the beam. From these results, plot a graph showing the distribution of shear stresses from top to bottom of the beam.



PROBLEM 5.8-6

5.8-7 A steel beam of length $L = 16$ in. and cross-sectional dimensions $b = 0.6$ in. and $h = 2$ in. (see figure) supports a uniform load of intensity $q = 240$ lb/in., which includes the weight of the beam.

Calculate the shear stresses in the beam (at the cross section of maximum shear force) at points located 1/4 in., 1/2 in., 3/4 in., and 1 in. from the top surface of the beam. From these calculations, plot a graph showing the distribution of shear stresses from top to bottom of the beam.



PROBLEM 5.8-7

Representative Problems

5.8-8 A beam of rectangular cross section (width b and height h) supports a uniformly distributed load along its entire length L. The allowable stresses in bending and shear are σ_{allow} and τ_{allow}, respectively.

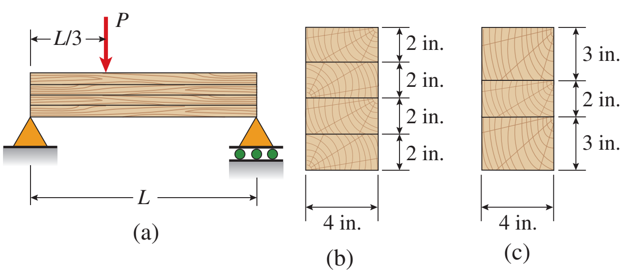
(a) If the beam is simply supported, what is the span length L_0 below which the shear stress governs the allowable load and above which the bending stress governs?

(b) If the beam is supported as a cantilever, what is the length L_0 below which the shear stress governs the allowable load and above which the bending stress governs?

5.8-9 A laminated wood beam on simple supports (figure part a) is built up by gluing together four 2 in. × 4 in. boards (actual dimensions) to form a solid beam 4 in. × 8 in. in cross section, as shown in the figure part b. The allowable shear stress in the glued joints is 62 psi, the allowable shear stress in the wood is 175 psi, and the allowable bending stress in the wood is 1650 psi.

(a) If the beam is 12 ft long, what is the allowable load P acting at the one-third point along the beam, as shown? (Include the effects of the beam's own weight, assuming that the wood weighs 35 lb/ft^3.)

(b) Repeat part (a) if the beam is assembled by gluing together two 3 in. × 4 in. boards and a 2 in. × 4 in. board (see figure part c).



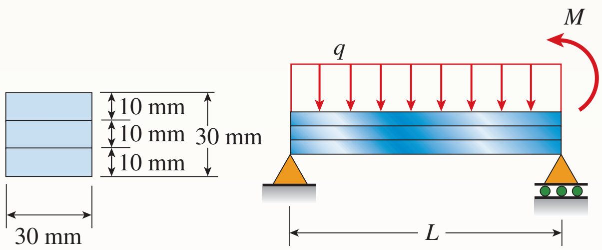
PROBLEM 5.8-9

5.8-10 A laminated plastic beam of square cross section is built up by gluing together three strips, each 10 mm × 30 mm in cross section (see figure). The beam has a total weight of 3.6 N and is simply supported with span length L = 360 mm.

Considering the weight of the beam (q), calculate the maximum permissible CCW moment M that may be placed at the right support.

(a) The allowable shear stress in the glued joints is 0.3 MPa.

(b) The allowable bending stress in the plastic is 8 MPa.

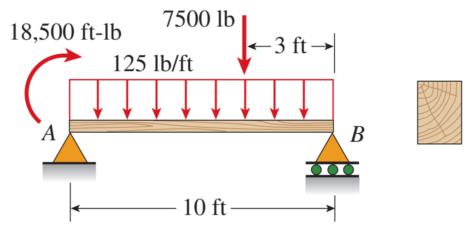


PROBLEM 5.8-10

5.8-11 A wood beam AB on simple supports with span length equal to 10 ft is subjected to a uniform load of intensity 125 lb/ft acting along the entire length of the beam, a concentrated load of magnitude 7500 lb acting at a point 3 ft from the right-hand support, and a moment at A of 18,500 ft-lb (see figure). The allowable stresses in bending and shear, respectively, are 2250 psi and 160 psi.

(a) From the table in Appendix G, select the lightest beam that will support the loads (disregard the weight of the beam).

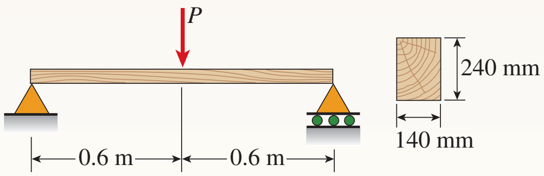
(b) Taking into account the weight of the beam (weight density = 35 lb/ft^3), verify that the selected beam is satisfactory, or if it is not, select a new beam.



PROBLEM 5.8-11

5.8-12 A simply supported wood beam of rectangular cross section and span length 1.2 m carries a concentrated load P at midspan in addition to its own weight (see figure). The cross section has width 140 mm and height 240 mm. The weight density of the wood is 5.4 kN/m^3.

Calculate the maximum permissible value of the load P if (a) the allowable bending stress is 8.5 MPa and (b) the allowable shear stress is 0.8 MPa.



PROBLEM 5.8-12

5.8-13 A square wood platform is 8 ft $\times$ 8 ft in area and rests on masonry walls (see figure). The deck of the platform is constructed of 2-in. nominal thickness tongue-and-groove planks (actual thickness 1.5 in.; see Appendix G) supported on two 8-ft long beams. The beams have 4 in. $\times$ 6 in. nominal dimensions (actual dimensions 3.5 in. $\times$ 5.5 in.).

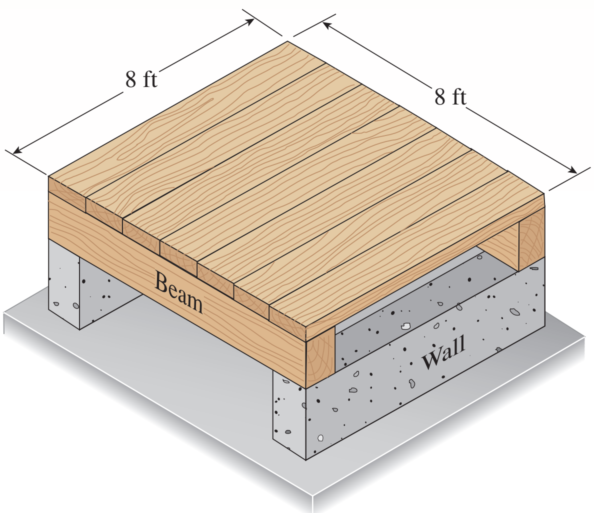
The planks are designed to support a uniformly distributed load w (lb/ft^2) acting over the entire top surface of the platform. The allowable bending stress for the planks is 2400 psi and the allowable shear stress is 100 psi. When analyzing the planks, disregard their weights and assume that their reactions are uniformly distributed over the top surfaces of the supporting beams.

(a) Determine the allowable platform load w_1 (lb/ft^2) based upon the bending stress in the planks.

(b) Determine the allowable platform load w_2 (lb/ft^2) based upon the shear stress in the planks.

(c) Which of the preceding values becomes the allowable load w_{allow} on the platform?

Hints: Use care in constructing the loading diagram for the planks, noting especially that the reactions are distributed loads instead of concentrated loads. Also, note that the maximum shear forces occur at the inside faces of the supporting beams.

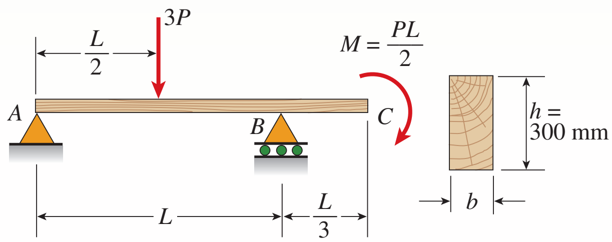


PROBLEM 5.8-13

5.8-14 A wood beam ABC with simple supports at A and B and an overhang BC has height $h = 300$ mm (see figure). The length of the main span of the beam is $L = 3.6$ m and the length of the overhang is $L/3 = 1.2$ m. The beam supports a concentrated load $3P = 18$ kN at the midpoint of the main span and a moment $PL/2 = 10.8$ kN $\cdot$ m at the free end of the overhang. The wood has a weight density $\gamma = 5.5$ kN/m^3.

(a) Determine the required width b of the beam based upon an allowable bending stress of 8.2 MPa.

(b) Determine the required width based upon an allowable shear stress of 0.7 MPa.



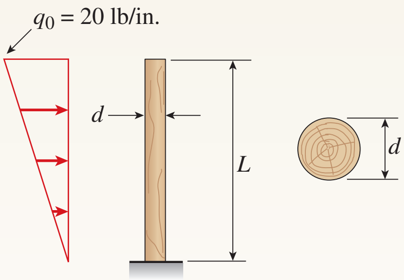
PROBLEM 5.8-14

5.9 Shear Stresses in Beams of Circular Cross Section

Introductory Problems

5.9-1 A wood pole with a solid circular cross section (d = diameter) is subjected to a triangular distributed horizontal force of peak intensity $q_0 = 20$ lb/in. (see figure). The length of the pole is $L = 6$ ft, and the allowable stresses in the wood are 1900 psi in bending and 120 psi in shear.

Determine the minimum required diameter of the pole based upon (a) the allowable bending stress, and (b) the allowable shear stress.

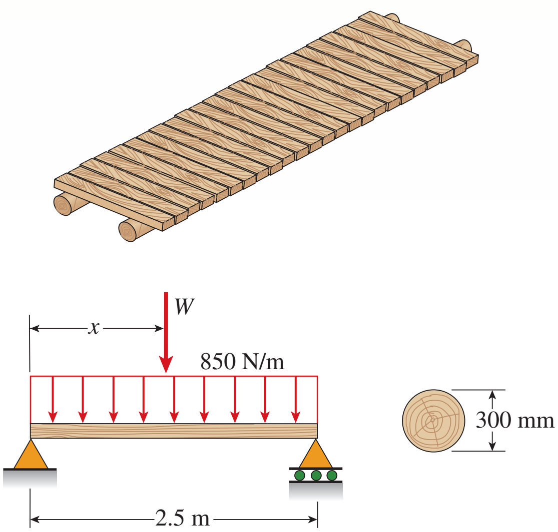


PROBLEM 5.9-1

5.9-2 A simple log bridge in a remote area consists of two parallel logs with planks across them (see figure). The logs are Douglas fir with an average diameter 300 mm. A truck moves slowly across the bridge, which spans 2.5 m. Assume that the weight of the truck is equally distributed between the two logs.

Because the wheelbase of the truck is greater than 2.5 m, only one set of wheels is on the bridge at a time. Thus, the wheel load on one log is equivalent to a concentrated load W acting at any position along the span. In addition, the weight of one log and the planks it supports is equivalent to a uniform load of 850 N/m acting on the log.

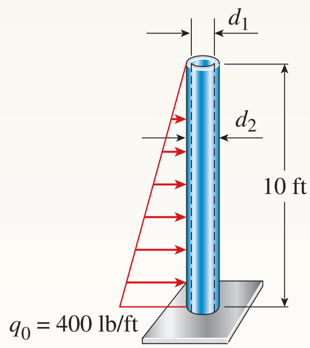
Determine the maximum permissible wheel load W based upon (a) an allowable bending stress of 7.0 MPa and (b) an allowable shear stress of 0.75 MPa.



PROBLEM 5.9-2

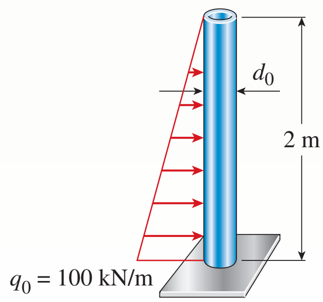
Representative Problems

5.9-3 A vertical pole consisting of a circular tube of outer diameter 5 in. and inner diameter 4.5 in. is loaded by a linearly varying distributed force with maximum intensity of q_0. Find the maximum shear stress in the pole.



PROBLEM 5.9-3

5.9-4 A circular pole is subjected to linearly varying distributed force with maximum intensity q_0. Calculate the diameter d_0 of the pole if the maximum allowable shear stress for the pole is 75 MPa.

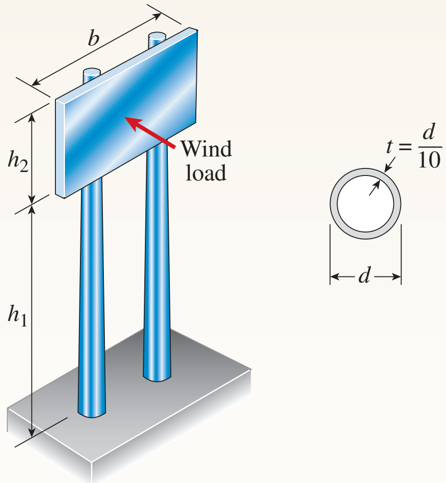


PROBLEM 5.9-4

5.9-5 A sign for an automobile service station is supported by two aluminum poles of hollow circular cross section, as shown in the figure. The poles are being designed to resist a wind pressure of 75 lb/ft^2 against the full area of the sign. The dimensions of the poles and sign are $h_1 = 20$ ft, $h_2 = 5$ ft, and $b = 10$ ft. To prevent buckling of the walls of the poles, the thickness t is specified as one-tenth the outside diameter d.

(a) Determine the minimum required diameter of the poles based upon an allowable bending stress of 7500 psi in the aluminum.

(b) Determine the minimum required diameter based upon an allowable shear stress of 2000 psi.

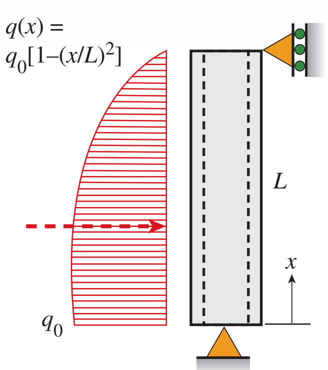


PROBLEM 5.9-5

5.9-6 A steel pipe is subjected to a quadratic distributed load over its height with the peak intensity q_0 at the base (see figure). Assume the following pipe properties and dimensions: height L, outside diameter $d = 200$ mm, and wall thickness $t = 10$ mm. Allowable stresses for flexure and shear are $\sigma_a = 125$ MPa and $\tau_a = 30$ MPa.

(a) If $L = 2.6$ m, find $q_{0,\max}$ (kN/m), assuming that allowable flexure and shear stresses in the pipe are not to be exceeded.

(b) If $q_0 = 60$ kN/m, find the maximum height $L_{\max}$ (m) of the pipe if the allowable flexure and shear stresses in the pipe are not to be exceeded.



PROBLEM 5.9-6

5.10 Shear Stresses in the Webs of Beams with Flanges

Introductory Problems

5.10-1 through 5.10-6 A wide-flange beam (see figure) is subjected to a shear force V. Using the dimensions of the cross section, calculate the moment of inertia and then determine the following quantities:

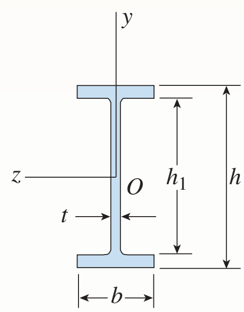
(a) The maximum shear stress $\tau_{\max}$ in the web.

(b) The minimum shear stress $\tau_{\min}$ in the web.

(c) The average shear stress τ_{aver} (obtained by dividing the shear force by the area of the web) and the ratio $\tau_{\max}/\tau_{\text{aver}}$.

(d) The shear force V_{web} carried in the web and the ratio V_{web}/V.

Note: Disregard the fillets at the junctions of the web and flanges and determine all quantities, including the moment of inertia, by considering the cross section to consist of three rectangles.



PROBLEMS 5.10-1 through 5.10-6

5.10-1 Dimensions of cross section: $b = 6$ in., $t = 0.5$ in., $h = 12$ in., $h_1 = 10.5$ in., and $V = 30$ k.

5.10-2 Dimensions of cross section: $b = 180$ mm, $t = 12$ mm, $h = 420$ mm, $h_1 = 380$ mm, and $V = 125$ kN.

5.10-3 Wide-flange shape, W 8 × 28 (see Table F-1, Appendix F); $V = 10$ k.

5.10-4 Dimensions of cross section: $b = 220$ mm, $t = 12$ mm, $h = 600$ mm, $h_1 = 570$ mm, and $V = 200$ kN.

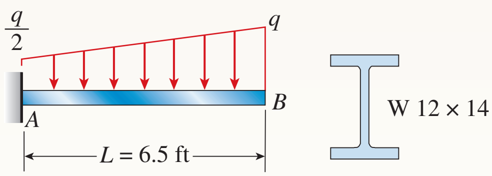
5.10-5 Wide-flange shape, W 18 × 71 (see Table F-1, Appendix F); $V = 21$ k.

5.10-6 Dimensions of cross section: $b = 120$ mm, $t = 7$ mm, $h = 350$ mm, $h_1 = 330$ mm, and $V = 60$ kN.

Representative Problems

5.10-7 A cantilever beam AB of length $L = 6.5$ ft supports a trapezoidal distributed load of peak intensity q, and minimum intensity $q/2$, that includes the weight of the beam (see figure). The beam is a steel W 12×14 wide-flange shape (see Table F-1(a), Appendix F).

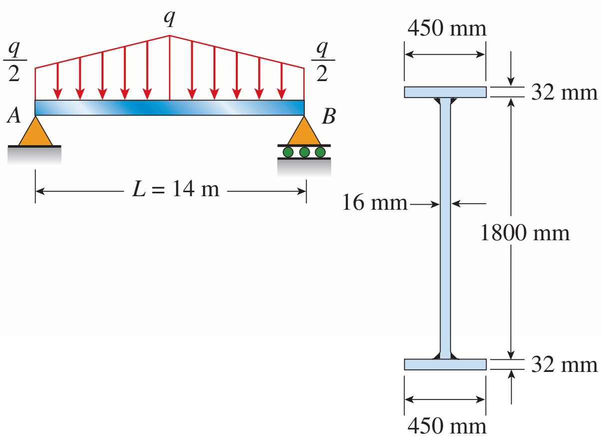
Calculate the maximum permissible load q based upon (a) an allowable bending stress $\sigma_{allow} = 18$ ksi and (b) an allowable shear stress $\tau_{allow} = 7.5$ ksi. *Note:* Obtain the moment of inertia and section modulus of the beam from Table F-1(a).



PROBLEM 5.10-7

5.10-8 A bridge girder AB on a simple span of length $L = 14$ m supports a distributed load of maximum intensity q at mid-span and minimum intensity $q/2$ at supports A and B that includes the weight of the girder (see figure). The girder is constructed of three plates welded to form the cross section shown.

Determine the maximum permissible load q based upon (a) an allowable bending stress $\sigma_{allow} = 110$ MPa and (b) an allowable shear stress $\tau_{allow} = 50$ MPa.

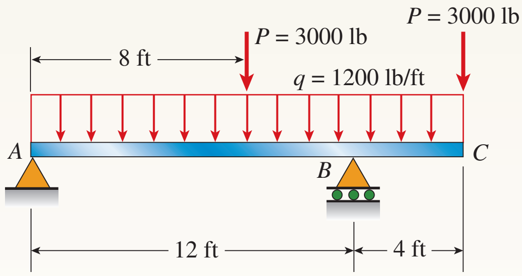


PROBLEM 5.10-8

5.10-9 A simple beam with an overhang supports a uniform load of intensity $q = 1200$ lb/ft and a concentrated $P = 3000$ lb load at 8 ft to the right of A and also at C (see figure). The uniform load includes an allowance for the weight of the beam. The allowable stresses in bending and shear are 18 ksi and 11 ksi, respectively.

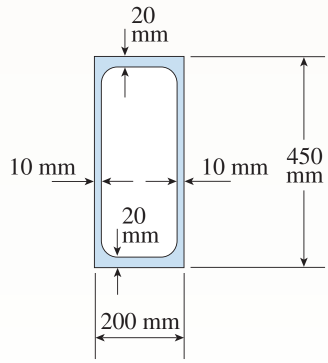
Select from Table F-2(a), Appendix F, the lightest I-beam (S shape) that will support the given loads.

Hint: Select a beam based upon the bending stress and then calculate the maximum shear stress. If the beam is overstressed in shear, select a heavier beam and repeat.



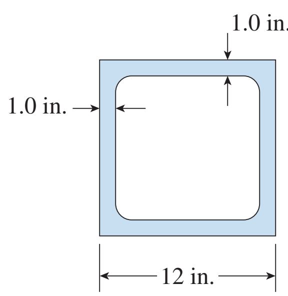
PROBLEM 5.10-9

5.10-10 A hollow steel box beam has the rectangular cross section shown in the figure. Determine the maximum allowable shear force V that may act on the beam if the allowable shear stress is 36 MPa.



PROBLEM 5.10-10

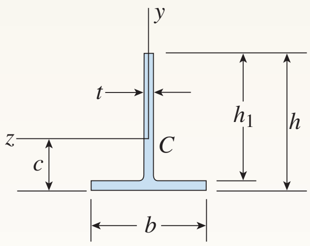
5.10-11 A hollow aluminum box beam has the square cross section shown in the figure. Calculate the maximum and minimum shear stresses τ_{max} and τ_{min} in the webs of the beam due to a shear force $V = 28$ k.



PROBLEM 5.10-11

5.10-12 The T-beam shown in the figure has cross-sectional dimensions: $b = 210$ mm, $t = 16$ mm, $h = 300$ mm, and $h_1 = 280$ mm. The beam is subjected to a shear force $V = 68$ kN.

Determine the maximum shear stress τ_{max} in the web of the beam.



PROBLEMS 5.10-12 and 5.10-13

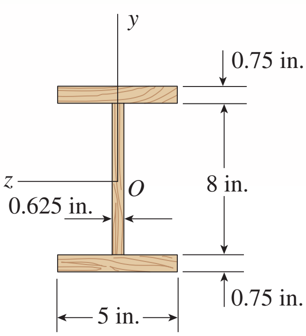
5.10-13 Calculate the maximum shear stress τ_{max} in the web of the T-beam shown in the figure if $b = 10$ in., $t = 0.5$ in., $h = 7$ in., $h_1 = 6.2$ in., and the shear force $V = 5300$ lb.

5.11 Built-Up Beams and Shear Flow

Introductory Problems

5.11-1 A prefabricated wood I-beam serving as a floor joist has the cross section shown in the figure. The allowable load in shear for the glued joints between the web and the flanges is 65 lb/in. in the longitudinal direction.

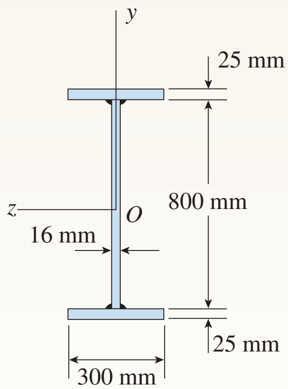
Determine the maximum allowable shear force V_{max} for the beam.



PROBLEM 5.11-1

5.11-2 A welded steel girder having the cross section shown in the figure is fabricated of two 300 mm × 25 mm flange plates and a 800 mm × 16 mm web plate. The plates are joined by four fillet welds that run continuously for the length of the girder. Each weld has an allowable load in shear of 920 kN/m.

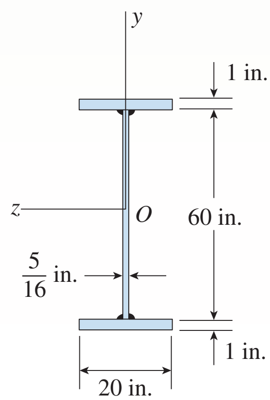
Calculate the maximum allowable shear force V_{max} for the girder.



PROBLEM 5.11-2

5.11-3 A welded steel girder having the cross section shown in the figure is fabricated of two 20 in. × 1 in. flange plates and a 60 in. × 5/16 in. web plate. The plates are joined by four longitudinal fillet welds that run continuously throughout the length of the girder.

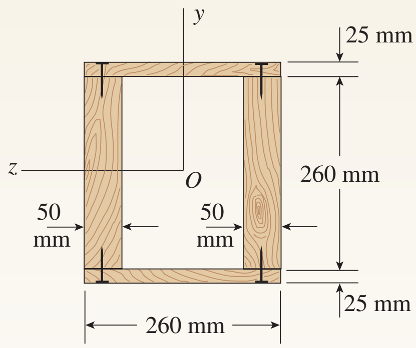
If the girder is subjected to a shear force of 280 kips, what force F (per inch of length of weld) must be resisted by each weld?



PROBLEM 5.11-3

5.11-4 A wood box beam is constructed of two 260 mm × 50 mm boards and two 260 mm × 25 mm boards (see figure). The boards are nailed at a longitudinal spacing $s = 100$ mm.

If each nail has an allowable shear force $F = 1200$ N, what is the maximum allowable shear force V_{max}?

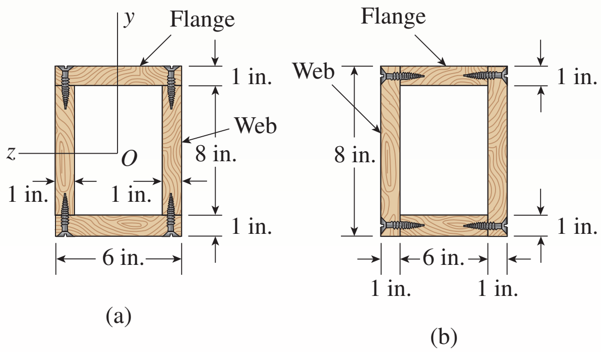


PROBLEM 5.11-4

5.11-5 A box beam is constructed of four wood boards as shown in the figure part a. The webs are 8 in. × 1 in. and the flanges are 6 in. × 1 in. boards (actual dimensions), joined by screws for which the allowable load in shear is $F = 250$ lb per screw.

(a) Calculate the maximum permissible longitudinal spacing s_{max} of the screws if the shear force V is 1200 lb.

(b) Repeat part (a) if the flanges are attached to the webs using a *horizontal arrangement* of screws as shown in the figure part b.



PROBLEM 5.11-5

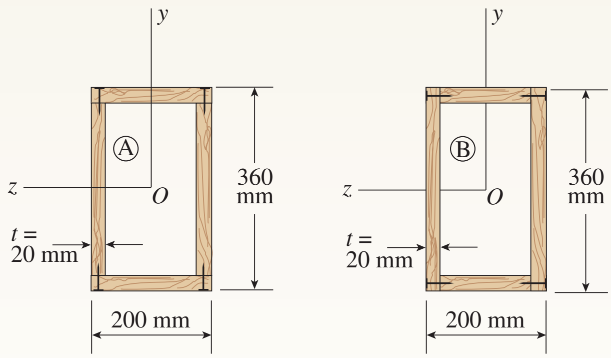
Representative Problems

5.11-6 Two wood box beams (beams A and B) have the same outside dimensions (200 mm × 360 mm) and the same thickness ($t = 20$ mm) throughout, as shown in the figure. Both beams are formed by nailing, with each nail having an allowable shear load of 250 N. The beams are designed for a shear force $V = 3.2$ kN.

(a) What is the maximum longitudinal spacing s_A for the nails in beam A?

(b) What is the maximum longitudinal spacing s_B for the nails in beam B?

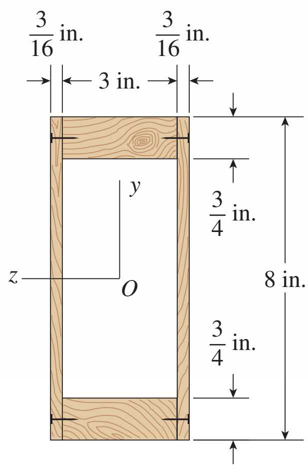
(c) Which beam is more efficient in resisting the shear force?



PROBLEM 5.11-6

5.11-7 A hollow wood beam with plywood webs has the cross-sectional dimensions shown in the figure. The plywood is attached to the flanges by means of small nails. Each nail has an allowable load in shear of 30 lb.

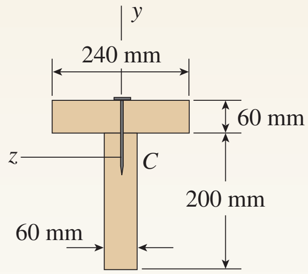
Find the maximum allowable spacing s of the nails at cross sections where the shear force V is equal to (a) 200 lb and (b) 300 lb.



PROBLEM 5.11-7

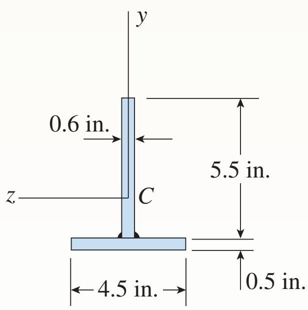
5.11-8 A beam of a T cross section is formed by nailing together two boards having the dimensions shown in the figure.

If the total shear force V acting on the cross section is 1500 N and each nail may carry 760 N in shear, what is the maximum allowable nail spacing s?



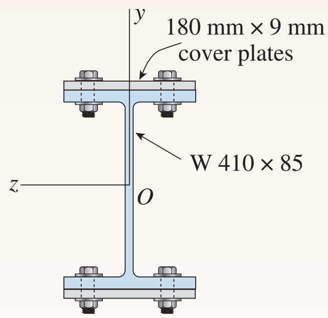
PROBLEM 5.11-8

5.11-9 The T-beam shown in the figure is fabricated by welding together two steel plates. If the allowable load for each weld is 1.8 kips/in. in the longitudinal direction, what is the maximum allowable shear force V?



PROBLEM 5.11-9

5.11-10 A steel beam is built up from a W 410 × 85 wide flange beam and two 180 mm × 9 mm cover plates (see figure). The allowable load in shear on each bolt is 9.8 kN. What is the required bolt spacing s in the longitudinal direction if the shear force $V = 110$ kN *Note:* Obtain the dimensions and moment of inertia of the W shape from Table F-1(b).

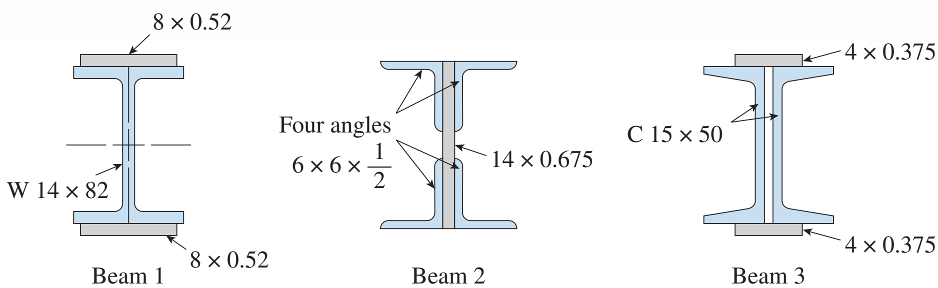


PROBLEM 5.11-10

5.11-11 The three beams shown have approximately the same cross-sectional area. Beam 1 is a W 14 × 82 with flange plates; beam 2 consists of a web plate with four angles; and beam 3 is constructed of 2 C shapes with flange plates.

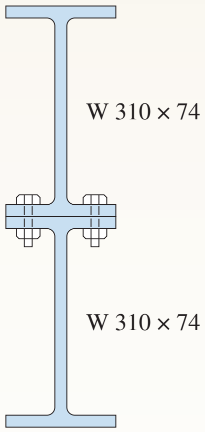
(a) Which design has the largest moment capacity?
(b) Which has the largest shear capacity?
(c) Which is the most economical in bending?
(d) Which is the most economical in shear?

Assume allowable stress values are: $\sigma_a = 18$ ksi and $\tau_a = 11$ ksi. The most economical beam is that having the largest capacity-to-weight ratio. Neglect fabrication costs in answering parts (c) and (d) above. *Note:* Obtain the dimensions and properties of all rolled shapes from tables in Appendix F.



PROBLEM 5.11-11

5.11-12 Two W 310 × 74 steel wide-flange beams are bolted together to form a built-up beam as shown in the figure. What is the maximum permissible bolt spacing s if the shear force $V = 80$ kN and the allowable load in shear on each bolt is $F = 13.5$ kN *Note:* Obtain the dimensions and properties of the W shapes from Table F-1(b).



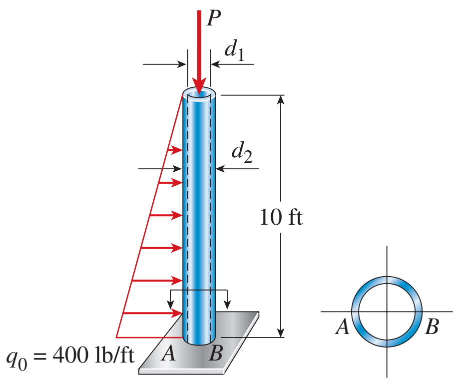
PROBLEM 5.11-12

5.12 Beams with Axial Loads

When solving the problems for Section 5.12, assume that the bending moments are not affected by the presence of lateral deflections.

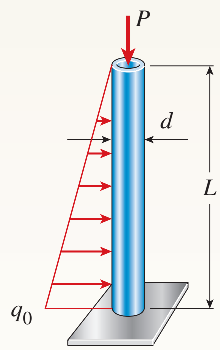
Introductory Problems

5.12-1 A pole is fixed at the base and is subjected to a linearly varying distributed force with maximum intensity of q_0 *and* an axial compressive load $P = 20$ kips at the top (see figure). The pole has a circular cross section with an outer diameter of 5 in. and an inner diameter of 4.5 in. Find the *normal* stresses on the surface of the pole at the base at locations A and B.



PROBLEM 5.12-1

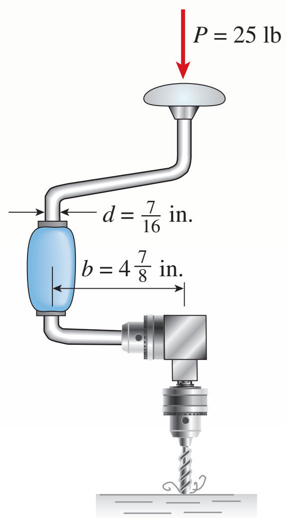
5.12-2 A solid circular pole is subjected to linearly varying distributed force with maximum intensity q_0 at the base and an axial compressive load P at the top (see figure). Find the required diameter d of the pole if the maximum allowable normal stress is 150 MPa. Let $q_0 = 6.5$ kN/m, $P = 70$ kN, and $L = 3$ m.



PROBLEM 5.12-2

5.12-3 While drilling a hole with a brace and bit, you exert a downward force $P = 25$ lb on the handle of the brace (see figure). The diameter of the crank arm is $d = 7/16$ in. and its lateral offset is $b = 4\text{-}7/8$ in.

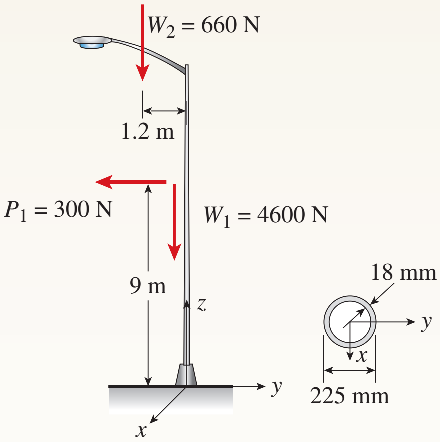
Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the crank.



PROBLEM 5.12-3

5.12-4 An aluminum pole for a street light weighs 4600 N and supports an arm that weighs 660 N (see figure). The center of gravity of the arm is 1.2 m from the axis of the pole. A wind force of 300 N also acts in the $(-y)$ direction at 9 m above the base. The outside diameter of the pole (at its base) is 225 mm, and its thickness is 18 mm.

Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the pole (at its base) due to the weights and the wind force.

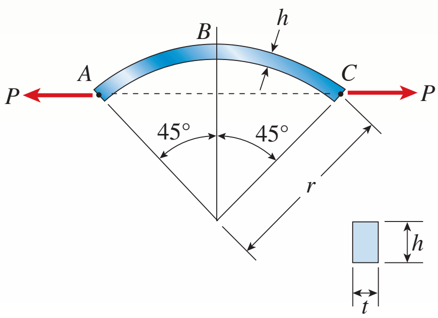


PROBLEM 5.12-4

Representative Problems

5.12-5 A curved bar ABC having a circular axis (radius $r = 12$ in.) is loaded by forces $P = 400$ lb (see figure). The cross section of the bar is rectangular with height h and thickness t.

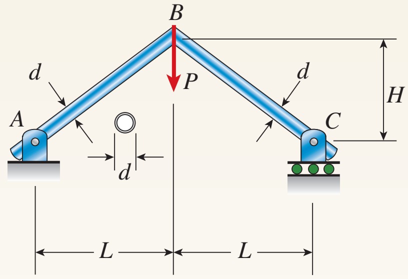
If the allowable tensile stress in the bar is 12,000 psi and the height $h = 1.25$ in., what is the minimum required thickness t_{min}?



PROBLEM 5.12-5

5.12-6 A rigid frame ABC is formed by welding two steel pipes at B (see figure). Each pipe has cross-sectional area $A = 11.31 \times 10^3$ mm^2, moment of inertia $I = 46.37 \times 10^6$ mm^4, and outside diameter $d = 200$ mm.

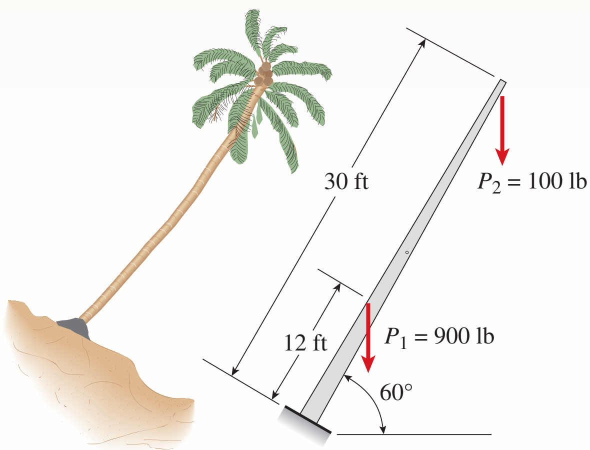
Find the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the frame due to the load $P = 8.0$ kN if $L = H = 1.4$ m.



PROBLEM 5.12-6

5.12-7 A palm tree weighing 1000 lb is inclined at an angle of 60° (see figure). The weight of the tree may be resolved into two resultant forces: a force $P_1 = 900$ lb acting at a point 12 ft from the base and a force $P_2 = 100$ lb acting at the top of the tree, which is 30 ft long. The diameter at the base of the tree is 14 in.

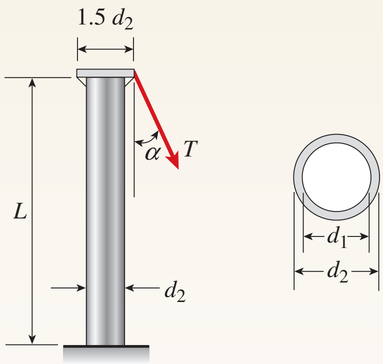
Calculate the maximum tensile and compressive stresses σ_t and σ_c, respectively, at the base of the tree due to its weight.



PROBLEM 5.12-7

5.12-8 A vertical pole of aluminum is fixed at the base and pulled at the top by a cable having a tensile force T (see figure). The cable is attached at the outer edge of a stiffened cover plate on top of the pole and makes an angle $\alpha = 20°$ at the point of attachment. The pole has length $L = 2.5$ m and a hollow circular cross section with an outer diameter $d_2 = 280$ mm and inner diameter $d_1 = 220$ mm. The circular cover plate has diameter $1.5d_2$.

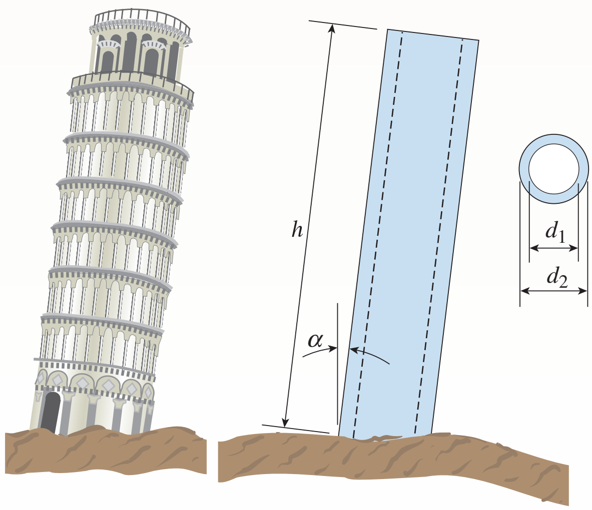
Determine the allowable tensile force T_{allow} in the cable if the allowable compressive stress in the aluminum pole is 90 MPa.



PROBLEM 5.12-8

5.12-9 Because of foundation settlement, a circular tower is leaning at an angle α to the vertical (see figure). The structural core of the tower is a circular cylinder of height h, outer diameter d_2, and inner diameter d_1. For simplicity in the analysis, assume that the weight of the tower is uniformly distributed along the height.

Obtain a formula for the maximum permissible angle α if there is to be no tensile stress in the tower.

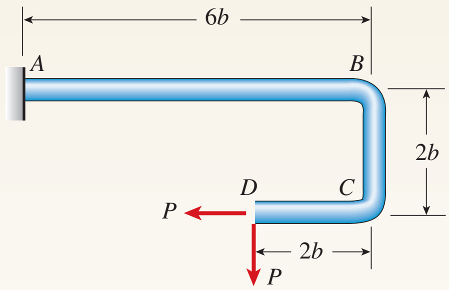


PROBLEM 5.12-9

5.12-10 A steel bracket of solid circular cross section is subjected to two loads, each of which is $P = 4.5$ kN at D (see figure). Let the dimension variable be $b = 240$ mm.

(a) Find the minimum permissible diameter d_{min} of the bracket if the allowable normal stress is 110 MPa.

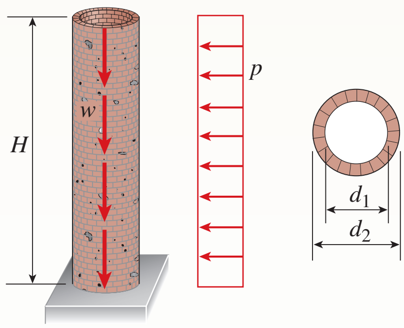
(b) Repeat part (a), including the weight of the bracket. The weight density of steel is 77.0 kN/m^3.



PROBLEM 5.12-10

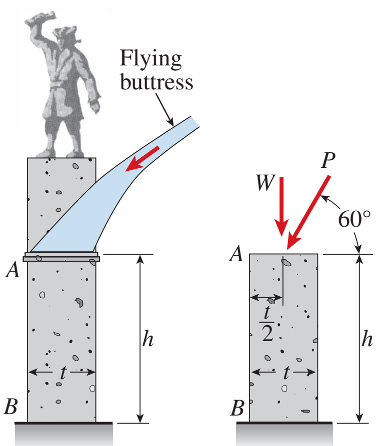
5.12-11 A cylindrical brick chimney of height H weighs $w = 825$ lb/ft of height (see figure). The inner and outer diameters are $d_1 = 3$ ft and $d_2 = 4$ ft, respectively. The wind pressure against the side of the chimney is $p = 10$ lb/ft^2 of projected area.

Determine the maximum height H if there is to be no tension in the brickwork.



PROBLEM 5.12-11

5.12-12 A flying buttress transmits a load $P = 25$ kN, acting at an angle of 60° to the horizontal, to the top of a vertical buttress AB (see figure). The vertical buttress has height $h = 5.0$ m and rectangular cross section of thickness $t = 1.5$ m and width $b = 1.0$ m

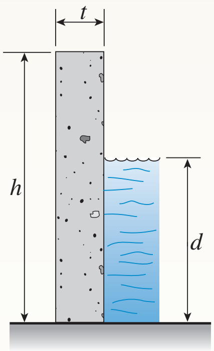


PROBLEM 5.12-12

(perpendicular to the plane of the figure). The stone used in the construction weighs $\gamma = 26$ kN/m^3.

What is the required weight W of the pedestal and statue above the vertical buttress (that is, above section A) to avoid any tensile stresses in the vertical buttress?

5.12-13 A plain concrete wall (i.e., a wall with no steel reinforcement) rests on a secure foundation and serves as a small dam on a creek (see figure). The height of the wall is $h = 6.0$ ft and the thickness of the wall is $t = 1.0$ ft.



PROBLEM 5.12-13

(a) Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, at the base of the wall when the water level reaches the top ($d = h$). Assume plain concrete has weight density $\gamma_c = 145$ lb/ft^3.

(b) Determine the maximum permissible depth d_{max} of the water if there is to be no tension in the concrete.

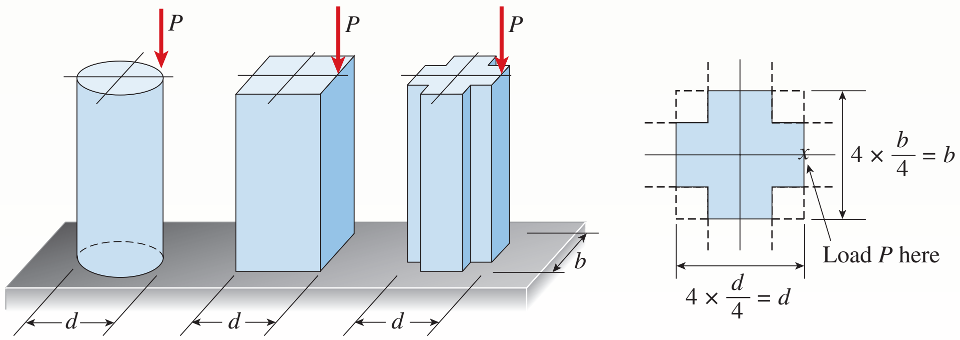
Eccentric Axial Loads

5.12-14 A circular post, a rectangular post, and a post of cruciform cross section are each compressed by loads that produce a resultant force P acting at the edge of the cross section (see figure). The diameter of the circular post and the depths of the rectangular and cruciform posts are the same.

(a) For what width b of the rectangular post will the maximum tensile stresses be the same in the circular and rectangular posts?

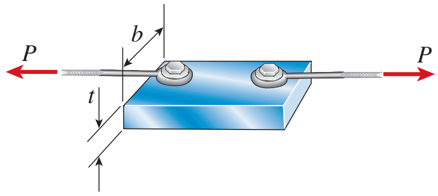
(b) Repeat part (a) for the post with cruciform cross section.

(c) Under the conditions described in parts (a) and (b), which post has the largest compressive stress?



PROBLEM 5.12-14

5.12-15 Two cables, each carrying a tensile force $P = 1200$ lb, are bolted to a block of steel (see figure). The block has thickness $t = 1$ in. and width $b = 3$ in.



PROBLEM 5.12-15

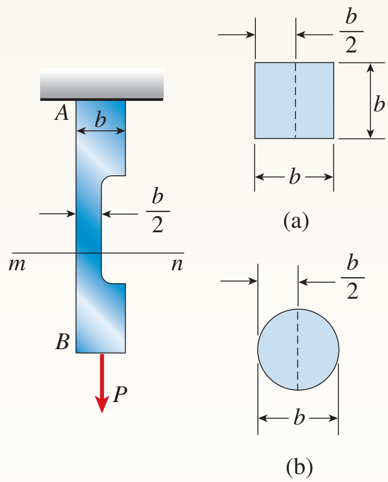
(a) If the diameter d of the cable is 0.25 in., what are the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the block?

(b) If the diameter of the cable is increased (without changing the force P), what happens to the maximum tensile and compressive stresses?

5.12-16 A bar AB supports a load P acting at the centroid of the end cross section (see figure). In the middle region of the bar the cross-sectional area is reduced by removing one-half of the bar.

(a) If the end cross sections of the bar are square with sides of length b, what are the maximum tensile and compressive stresses σ_t and σ_c, respectively, at cross section *mn* within the reduced region?

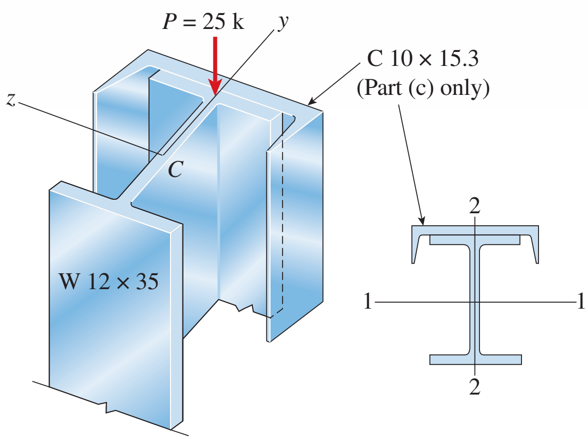
(b) If the end cross sections are circular with diameter b, what are the maximum stresses σ_t and σ_c?



PROBLEM 5.12-16

5.12-17 A short column constructed of a W 12 × 35 wide-flange shape is subjected to a resultant compressive load $P = 25$ k having its line of action at the midpoint of one flange (see figure).

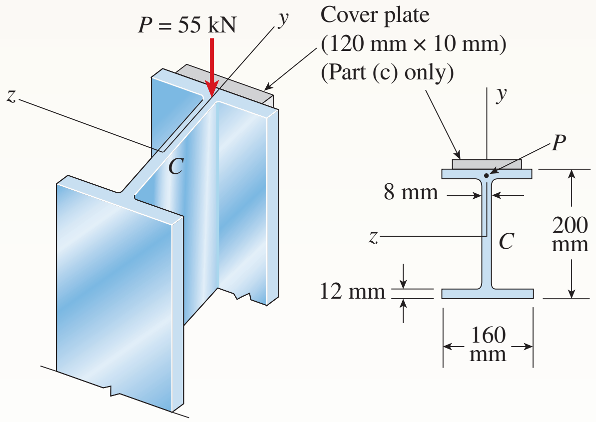
(a) Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the column.
(b) Locate the neutral axis under this loading condition.
(c) Recompute maximum tensile and compressive stresses if a C 10 × 15.3 is attached to one flange, as shown.



PROBLEM 5.12-17

5.12-18 A short column with a wide-flange shape is subjected to a compressive load that produces a resultant force $P = 55$ kN acting at the midpoint of one flange (see figure).

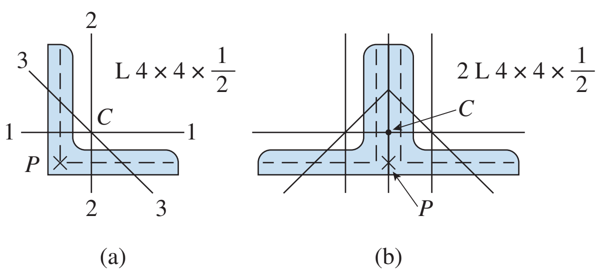
(a) Determine the maximum tensile and compressive stresses σ_t and σ_c, respectively, in the column.
(b) Locate the neutral axis under this loading condition.
(c) Recompute maximum tensile and compressive stresses if a 120 mm × 10 mm cover plate is added to one flange as shown.



PROBLEM 5.12-18

5.12-19 A tension member constructed of an L 4 × 4 × $\frac{1}{2}$ inch angle section (see Table F-4(a) in Appendix F) is subjected to a tensile load $P = 12.5$ kips that acts through the point where the mid-lines of the legs intersect (see figure part a).

(a) Determine the maximum tensile stress σ_t in the angle section.
(b) Recompute the maximum tensile stress if two angles are used and P is applied as shown in the figure part b.



PROBLEM 5.12-19

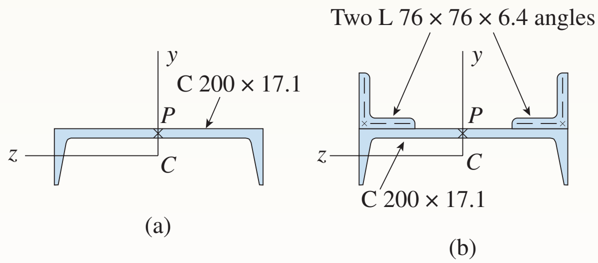
5.12-20 A short length of a C 200 × 17.1 channel is subjected to an axial compressive force P that has its line of action through the midpoint of the web of the channel (see figure part a).

(a) Determine the equation of the neutral axis under this loading condition.

(b) If the allowable stresses in tension and compression are 76 MPa and 52 MPa respectively, find the maximum permissible load P_{max}.

(c) Repeat parts (a) and (b) if two L 76 × 76 × 6.4 angles are added to the channel as shown in the figure part b.

See Table F-3(b) in Appendix F for channel properties and Table F-4(b) for angle properties.

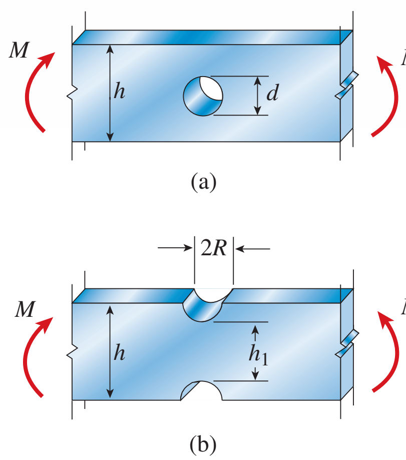


PROBLEM 5.12-20

5.13 Stress Concentrations in Bending

The problems for Section 5.13 are to be solved considering the stress-concentration factors.

5.13-1 The beams shown in the figure are subjected to bending moments $M = 2100$ lb-in. Each beam has a rectangular cross section with height $h = 1.5$ in. and width $b = 0.375$ in. (perpendicular to the plane of the figure).



PROBLEMS 5.13-1 through 5.13-4

(a) For the beam with a hole at midheight, determine the maximum stresses for hole diameters $d = 0.25$, 0.50, 0.75, and 1.00 in.

(b) For the beam with two identical notches (inside height $h_1 = 1.25$ in.), determine the maximum stresses for notch radii $R = 0.05$, 0.10, 0.15 and 0.20 in.

5.13-2 The beams shown in the figure are subjected to bending moments $M = 250$ N · m. Each beam has a rectangular cross section with height $h = 44$ mm and width $b = 10$ mm (perpendicular to the plane of the figure).

(a) For the beam with a hole at midheight, determine the maximum stresses for hole diameters $d = 10$, 16, 22, and 28 mm.

(b) For the beam with two identical notches (inside height $h_1 = 40$ mm), determine the maximum stresses for notch radii $R = 2$, 4, 6, and 8 mm.

5.13-3 A rectangular beam with semicircular notches, as shown in part b of the figure, has dimensions $h = 0.88$ in. and $h_1 = 0.80$ in. The maximum allowable bending stress in the metal beam is $\sigma_{max} = 60$ ksi, and the bending moment is $M = 600$ lb-in.

Determine the minimum permissible width b_{min} of the beam.

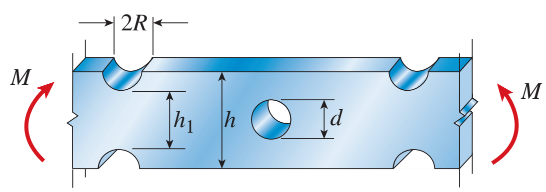
5.13-4 A rectangular beam with semicircular notches, as shown in part b of the figure, has dimensions $h = 120$ mm and $h_1 = 100$ mm. The maximum allowable bending stress in the plastic beam is $\sigma_{max} = 6$ MPa, and the bending moment is $M = 150$ N · m.

Determine the minimum permissible width b_{min} of the beam.

5.13-5 A rectangular beam with notches and a hole (see figure) has dimensions $h = 5.5$ in., $h_1 = 5$ in., and width $b = 1.6$ in. The beam is subjected to a bending moment $M = 130$ kip-in., and the maximum allowable bending stress in the material (steel) is $\sigma_{max} = 42{,}000$ psi.

(a) What is the smallest radius R_{min} that should be used in the notches?

(b) What is the diameter d_{max} of the largest hole that should be drilled at the midheight of the beam?



PROBLEM 5.13-5